Name ______________________ Date ____________

BENCHMARK 4

(Chapters 7 and 8)

A. Solving Linear Systems by Graphing (pp. 58–62)

A linear system can have zero, one, or infinitely many solutions. One way to find this solution is to graph each equation in the system. The point where the lines intersect is the solution to the system. If the graph of the equations are parallel lines, then no point of intersection exists, and the system has no solution. If each equation represents the same line, the system has infinitely many solutions.

1. Checking Solutions to Linear Systems

Vocabulary

System of linear equations Two or more linear equations in the same variables; also called a *linear system.*

EXAMPLE **Tell whether the ordered pair is a solution of the linear system.**

For a linear system with one solution, the solution is an ordered pair that satisfies each equation in the system.

a. (2, 1);

$2x - y = 3$ **Equation 1**

$-x + 3y = 1$ **Equation 2**

b. (0, 5);

$4x - 2y = -10$ **Equation 1**

$3x + y = 15$ **Equation 2**

Solution:

a. Substitute 2 for x and 1 for y in each equation.

$2x - y = 3$	$-x + 3y = 1$
$2(2) - (1) \stackrel{?}{=} 3$	$-2 + 3(1) \stackrel{?}{=} 1$
$4 - 1 \stackrel{?}{=} 3$	$-2 + 3 \stackrel{?}{=} 1$
$3 = 3$ ✔	$1 = 1$ ✔

The ordered pair (2, 1) is a solution of each equation, so it is the solution of the system.

b. Substitute 0 for x and 5 for y in each equation.

$4x - 2y = -10$	$3x + y = 15$
$4(0) - 2(5) \stackrel{?}{=} -10$	$3(0) + 5 \stackrel{?}{=} 15$
$0 - 10 \stackrel{?}{=} -10$	$0 + 5 \stackrel{?}{=} 15$
$-10 = -10$ ✔	$5 = 15$ ✗

The ordered pair (0, 5) is NOT the solution of $3x + y = 15$, so it is NOT the solution of the system.

PRACTICE **Tell whether the ordered pair is a solution of the linear system.**

1. (4, −2);
$3x + 3y = 6$
$-2x + 4y = -16$

2. (−3, 3);
$x + 2y = 3$
$-2x - 3y = -3$

3. (−1, 0);
$6x + 4y = -6$
$5x - 2y = 2$

4. (3, 6);
$2x - 3y = 12$
$-x + 5y = 27$

5. (2, −8);
$10x + 5y = -20$
$-3x - y = -2$

6. (1, 5);
$4x - 2y = -6$
$-3x + 4y = 17$

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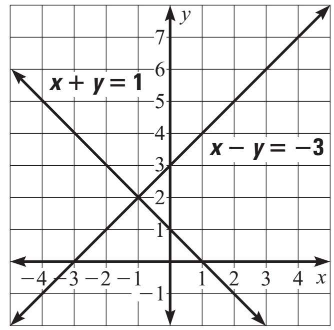
2. Graphing Linear Systems

EXAMPLE **Use the graph to solve the system. Then check your solution algebraically.**

$x - y = -3$ **Equation 1**

$x + y = 1$ **Equation 2**



Solution:

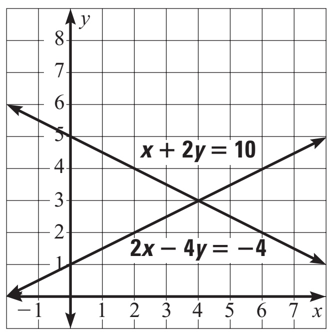
The lines seem to intersect at $(-1, 2)$. Check by substituting -1 for x and 2 for y in each equation.

$x - y = -3$	$x + y = 1$
$-1 - 2 \stackrel{?}{=} -3$	$-1 + 2 \stackrel{?}{=} 1$
$-3 = -3$ ✔	$1 = 1$ ✔

The ordered pair $(-1, 2)$ is the solution of each equation. So, $(-1, 2)$ is the solution of the system.

PRACTICE **Use the graph to solve the system. Then check your solution algebraically.**

7.



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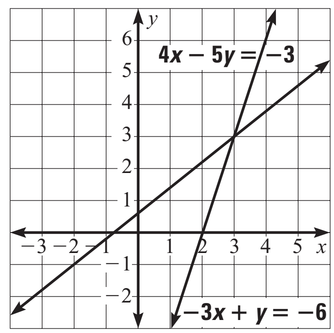

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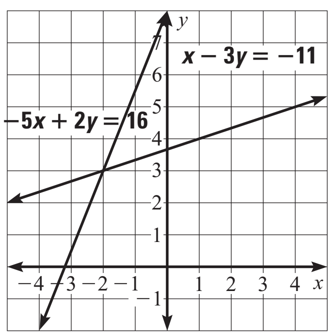

3. Graph-and-Check Method

Vocabulary **Consistent independent system** A linear system with exactly one solution.

EXAMPLE **Solve the linear system:**

$3x - y = 2$ **Equation 1**

$x + 2y = 10$ **Equation 2**

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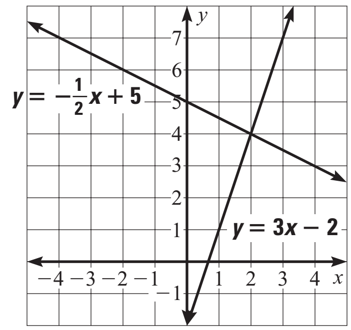
Solution:

Step 1: Graph both equations.



Step 2: Estimate the point of intersection. The two lines appear to intersect at (2, 4).

Step 3: Check whether (2, 4) is a solution by substituting 2 for x and 4 for y in each of the original equations.

Equation 1	**Equation 2**
$3x - y = 2$	$x + 2y = 10$
$3(2) - 4 \stackrel{?}{=} 2$	$2 + 2(4) \stackrel{?}{=} 10$
$2 = 2$ ✔	$10 = 10$ ✔

Because (2, 4) is a solution of each equation, it is a solution of the linear system.

PRACTICE **Solve the linear system by graphing. Check your solution.**

10. $3x + y = 5$
$5x - 2y = 12$

11. $4x - 2y = -10$
$2x + y = 1$

12. $-2x + y = -9$
$x - 3y = 12$

13. $2x - y = -2$
$-3x + 3y = 9$

14. $6x - 4y = 4$
$x - y = 0$

15. $2x + 2y = 6$
$-x + 2y = -9$

4. Special Types of Linear Systems

Vocabulary

Inconsistent system A linear system with no solution; the graphs of the equations are parallel.

Consistent dependent system A linear system with infinitely many solutions; the graphs of the equations are the same line.

EXAMPLE **Show that the linear system has no solution.**

$2x - 3y = -3$ **Equation 1**
$2x - 3y = 15$ **Equation 2**

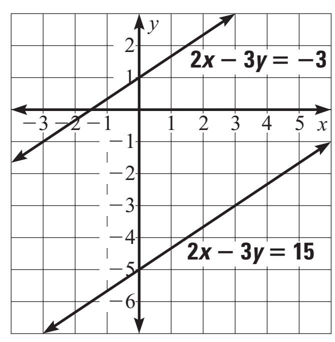
Solution:

You can use either of two methods to solve the problem.

Method 1: Graphing

Graph the linear system.

The lines have the same slope, but different y-intercepts. So, the equations represent two parallel lines, which do not intersect. The system has no solution.



Lines that do not intersect are said to be *inconsistent*. So, a linear system with no solutions is an inconsistent system.

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Method 2: Elimination

Subtract the equations.

$$\begin{array}{r} 2x - 3y = -3 \\ -\ (\underline{2x - 3y = 15}) \end{array} \longrightarrow \begin{array}{r} 2x - 3y = -3 \\ \underline{-2x + 3y = -15} \\ 0 = -18 \text{ ✗} \end{array}$$

← **This is a false statement.**

Subtracting the equations leads to a false statement, so the system has no solution.

EXAMPLE **Show that the linear system has infinitely many solutions.**

$y = \frac{3}{4}x - 2$ **Equation 1**

$3x - 4y = 8$ **Equation 2**

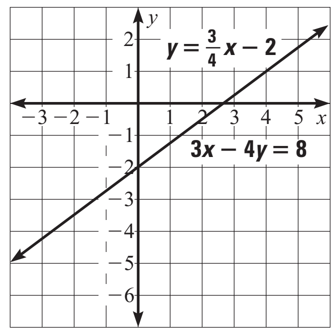
Solution:

You can use either of two methods to solve the problem.

Method 1: Graphing

Graph the linear system.

Both equations represent the same line. So, all points on the line are solutions. The system has infinitely many solutions.



Lines that intersect are said to be *consistent,* and equations that are equivalent are said to be *dependent*. So, a linear system in which all the equations represent the same line is a consistent, dependent system.

Method 2: Substitution

Substitute $\frac{3}{4}x - 2$ for y in Equation 2 and solve for x.

$3x - 4y = 8$ **Write Equation 2.**

$3x - 4\left(\frac{3}{4}x - 2\right) = 8$ **Substitute $\frac{3}{4}x - 2$ for y.**

$3x - 3x + 8 = 8$ **Simplify.**

$8 = 8$ **Simplify.**

Substitution leads to a statement that is always true. The system has infinitely many solutions.

PRACTICE **Tell whether the linear system has *no solution* or *infinitely many solutions.* Explain.**

16. $x - 4y = -20$
$x - 4y = 8$

17. $y = -\frac{3}{4}x - 6$
$3x + 4y = -24$

18. $3x - 2y = -8$
$-3x + 2y = -6$

19. $3x + \frac{1}{3}y = 1$
$9x + y = -6$

20. $y = -5x + 3$
$10x + 2y = 6$

21. $14y - 6x = -28$
$3x - 7y = 14$

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Quiz

Tell whether the ordered pair is a solution of the linear system.

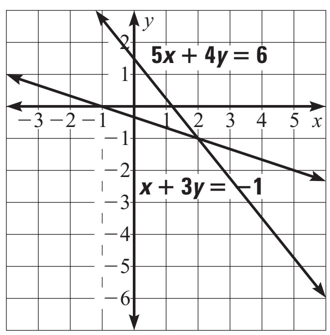
1. $(-3, 2)$;
$x - 2y = -7$
$3x - 2y = -13$

2. $(-7, -5)$;
$x + y = -12$
$2x - 4y = -34$

3. $(9, -10)$;
$2x + y = 8$
$-3x + 4y = -67$

Use the graph to solve the system. Then check your solution algebraically.

4.



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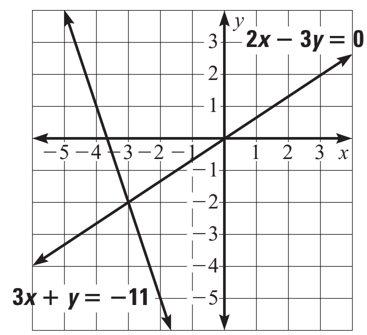

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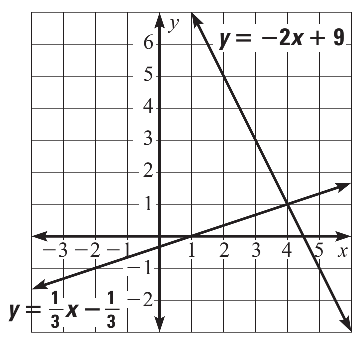

Solve the linear system. If the system has one solution, check the solution. If the system has no solution or infinitely many solutions, explain.

7. $x + 4y = 2$
$3x - 5y = 6$

8. $3x + y = -4$
$y = -3x + 1$

9. $8x - 6y = -2$
$x + 3y = -4$

10. $x - 4y = -3$
$-2x + 6y = 2$

11. $2x + y = 1$
$5x + 3y = 0$

12. $2y = -x + 8$
$2x + 4y = 16$