

Benchmark 4

(Chapters 7 and 8)

B. Solving Linear Systems Using Algebra (pp. 63–66)

A **system of linear equations** is two or more linear equations with the same variables. The **solution of a system of linear equations** is an ordered pair that satisfies each equation in the system. The following examples describe methods which can be used to find the solution of a system of two linear equations.

1. Solve a Linear System by Substitution

Vocabulary

Substitution Replacing a variable in an equation by an expression or a value.

EXAMPLE

Solve the linear system:

$$2x - 3 = y \quad \text{Equation 1}$$

$$3x - 2y = 4 \quad \text{Equation 2}$$

Solution:

Step 1: Solve for a variable in one of the equations.

$$y = 2x - 3 \quad \text{Equation 1 is already solved for } y.$$

Step 2: Substitute the expressions from Step 1 into the other equation and solve.

$$3x - 2y = 4 \quad \text{Write Equation 2.}$$

$$3x - 2(2x - 3) = 4 \quad \text{Substitute } 2x - 3 \text{ for } y \text{ into Equation 2.}$$

$$3x - 4x + 6 = 4 \quad \text{Simplify.}$$

$$-x = -2 \quad \text{Combine like terms.}$$

$$x = 2 \quad \text{Multiply each side by } -1.$$

Step 3: Use the value found in Step 2 to find the value of the other variable.

$$y = 2x - 3 = 2(2) - 3 = 1 \quad \text{Substitute } x = 2 \text{ into Equation 1.}$$

The solution is (2, 1).

Don't stop after solving for one variable. The system is not solved until a solution is found for both variables.

PRACTICE

Use the substitution method to solve the linear system.

- | | | |
|-----------------------------------|----------------------------------|----------------------------------|
| 1. $y = 2x + 5$
$3x + y = 10$ | 2. $x = y + 3$
$x + 2y = -6$ | 3. $2x + 4y = 0$
$3x + 7 = y$ |
| 4. $x + 2y = -5$
$4x - 3y = 2$ | 5. $3x - 2y = 4$
$x + 3y = 5$ | 6. $3x + y = -2$
$x + 3y = 2$ |

2. Solve a Linear System by Adding or Subtracting

Vocabulary

Elimination Performing operations on and combining two or more equations in a system in such a way that one of the variables is no longer present.

BENCHMARK 4*(Chapters 7 and 8)***EXAMPLE Solve the linear system:**

$$2x + 3y = 9 \quad \text{Equation 1}$$

$$2y - 2x = -4 \quad \text{Equation 2}$$

Check solutions by substituting them into the original equations to see if the resulting statements are true.

Solution:

Step 1: Add the equations to eliminate one variable and solve.

$$\begin{array}{r} 2x + 3y = 9 \\ + -2x + 2y = -4 \\ \hline 5y = 5 \\ y = 1 \end{array}$$

Add Equation 1 and Equation 2.

Solve for y.

Step 2: Use the value found in Step 1 to find the value of the other variable.

$$2x + 3y = 9$$

Write Equation 1.

$$2x + 3(1) = 9$$

Substitute $y = 1$ into Equation 1.

$$2x = 6$$

Simplify.

$$x = 3$$

Divide each side by 2.

The solution is (3, 1).

EXAMPLE Solve the linear system:

$$4x + 3y = 3 \quad \text{Equation 1}$$

$$9 = 2x + 3y \quad \text{Equation 2}$$

Solution:

Step 1: Subtract the equations to eliminate one variable and solve.

$$\begin{array}{r} 4x + 3y = 3 \\ - 2x + 3y = 9 \\ \hline 2x = -6 \\ x = -3 \end{array}$$

Subtract Equation 1 and Equation 2.

Solve for x.

Step 2: Use the value found in Step 1 to find the value of the other variable.

$$4x + 3y = 3$$

Write Equation 1.

$$4(-3) + 3y = 3$$

Substitute $x = -3$ into Equation 1.

$$3y = 15$$

Simplify.

$$y = 5$$

Divide each side by 3.

The solution is (-3, 5).

When subtracting equations, make sure each term of the second equation is subtracted from each like term in the first equation.

Benchmark 4*(Chapters 7 and 8)***PRACTICE****Use the method of adding or subtracting equations to solve the linear system.**

7. $2x = 3y$
 $x + 3y = 9$

8. $4x + 3y = 8$
 $-y + 4x = -8$

9. $5x - 2y = 20$
 $3x + 4 = -2y$

10. $3y - 4x = 3$
 $-x + 3y = -15$

11. $3x - 7 = 5y$
 $3x + 4y = -11$

12. $-2x = y - 3$
 $7x + y = 3$

3. Solve a Linear System by Multiplying First**EXAMPLE****Solve the linear system:**

$-2x + 4y = 2$ Equation 1

$-5x + 6y = -3$ Equation 2

Solution:

Step 1: Modify the original system by multiplying both equations by a constant. Use the least common multiple of the coefficients for one of the variables to determine the constants.

$$\begin{array}{l} -2x + 4y = 2 \quad (\times 3) \quad \rightarrow \quad -6x + 12y = 6 \\ -5x + 6y = -3 \quad (\times -2) \quad \rightarrow \quad 10x - 12y = 6 \end{array}$$

The least common multiple of 4 and 6 is 12, so multiply Equation 1 by 3 to get $12y$ and Equation 2 by -2 to get $-12y$.

Step 2: Add the modified equations from Step 1 and solve.

$$\begin{array}{r} -6x + 12y = 6 \\ + 10x - 12y = 6 \\ \hline 4x = 12 \\ x = 3 \end{array}$$

Add the modified equations.

Solve for x .

Step 3: Use the value found in Step 2 to find the value of the other variable.

$$\begin{array}{l} -2x + 4y = 2 \\ -2(3) + 4y = 2 \\ 4y = 8 \\ y = 2 \end{array}$$

Write Equation 1.

Substitute $x = 3$ into Equation 1.

Simplify.

Divide each side by 4.

The solution is $(3, 2)$.**PRACTICE****Multiply one or both equations by constants to solve the linear system.**

13. $2x + 12 = 2y$
 $3x + y = -10$

14. $5x + 2y = 14$
 $2x - 3y = -2$

15. $5y - 4x = -1$
 $6x - 7y = 3$

16. $3y - 2x = -10$
 $4x + 7y = -6$

17. $6y - 5x = 8$
 $7x - 8y = -12$

18. $3x - 2y = -3$
 $9x - 4y = 3$

BENCHMARK 4*(Chapters 7 and 8)***Quiz**

Use the substitution method to find the solution to the set of equations.

1. $x + 3y = 10$
 $2x - y = 6$

2. $5x = y - 1$
 $3x + 9 = 2y$

3. $5y - 2x = -4$
 $x - 3y = 3$

Use the elimination method to find the solution to the set of equations.

4. $4x = 3y - 6$
 $x + 3y = -9$

5. $2x + 5y = 3$
 $2x + 7y = 5$

6. $4y - 5x = 3$
 $3x = 4y - 5$

7. $2x - 3y = 0$
 $3x - 4y = 1$

8. $4y - 3x = -1$
 $5x - 2y = -3$

9. $3x + 2y = 2$
 $6x + 5y = 8$

For each linear system, name the method which is most appropriate for solving the system: substitution, adding equations, subtracting equations, or multiplying first. Then solve the system.

10. $2y = 5x + 3$
 $-3x + 2y = 5$

11. $4x = 3y - 4$
 $3x - y = 2$

12. $5x + 6y = -4$
 $4x + 3y = -5$

13. You have 50 tickets to ride the Ferris wheel and the roller coaster. If you ride 12 times total, using 3 tickets for each Ferris wheel ride and 5 tickets for each roller coaster ride, how many times did you go on each ride?