Name _____ Date _____

BENCHMARK 4

(Chapters 7 and 8)

D. Properties of Exponents (pp. 70–73)

A **power** represents repeated multiplication. The following examples illustrate several properties that you can use to simplify expressions with powers.

1. Product of Powers Property

Vocabulary

Product of Powers Property To multiply powers having the same base, add the exponents.

EXAMPLE

Simplify the expression.

a. $4^2 \cdot 4^8$ **b.** $3 \cdot 3^7 \cdot 3^6$ **c.** $(-12)(-12)^4$ **d.** $x^2 \cdot x^{10} \cdot x^5$

If a is a real number and m and n are positive integers, $a^m \cdot a^n = a^{m+n}$.

Solution:

a. $4^2 \cdot 4^8 = 4^{2+8} = 4^{10}$

b. $3 \cdot 3^7 \cdot 3^6 = 3^1 \cdot 3^7 \cdot 3^6 = 3^{1+7+6} = 3^{14}$

c. $(-12)(-12)^4 = (-12)^1 \cdot (-12)^4 = (-12)^{1+4} = (-12)^5$

d. $x^2 \cdot x^{10} \cdot x^5 = x^{2+10+5} = x^{17}$

PRACTICE

Simplify the expression.

1. $8^7 \cdot 8^2$ **2.** $5^3 \cdot 5 \cdot 5^9$ **3.** $(-6)^7(-6)^4(-6)$

4. $d^{10} \cdot d^7$ **5.** $r^5 \cdot r^7 \cdot r^8$ **6.** $k^2 \cdot k^3 \cdot k$

2. Power of a Power Property

Vocabulary

Power of a Power Property To find a power of a power, multiply exponents.

EXAMPLE

Simplify the expression.

a. $(3^2)^4$ **b.** $[(-4)^9]^2$ **c.** $(x^3)^3$ **d.** $[(y-3)^5]^4$

If a is a real number and m and n are positive integers, $(a^m)^n = a^{mn}$.

Solution:

a. $(3^2)^4 = 3^{2 \cdot 4} = 3^8$

b. $[(-4)^9]^2 = (-4)^{9 \cdot 2} = (-4)^{18}$

c. $(x^3)^3 = x^{3 \cdot 3} = x^9$

d. $[(y-3)^5]^4 = (y-3)^{5 \cdot 4} = (y-3)^{20}$

PRACTICE

Simplify the expression.

7. $(10^3)^6$ **8.** $[(-6)^2]^8$ **9.** $(t^5)^3$

10. $(b^4)^4$ **11.** $[(p+5)^7]^2$ **12.** $[(h-1)^9]^5$

Name _____ Date _____

Benchmark 4

(Chapters 7 and 8)

3. Power of a Product Property

Vocabulary

Power of a Product Property To find a power of a product, find the power of each factor and multiply.

EXAMPLE

Evaluate the numerical power when simplifying powers with both numerical and variable bases.

Simplify the expression.

a. $(9 \cdot 7)^3$ **b.** $(5xy)^2$ **c.** $(-3z)^4$ **d.** $-(3z)^4$

Solution:

a. $(9 \cdot 7)^3$
$= 9^3 \cdot 7^3$

b. $(5xy)^2$
$= (5 \cdot x \cdot y)^2$
$= 5^2 \cdot x^2 \cdot y^2$
$= 25x^2y^2$

c. $(-3z)^4$
$= (-3 \cdot z)^4$
$= (-3)^4 \cdot z^4$
$= 81z^4$

d. $-(3z)^4$
$= -(3 \cdot z)^4$
$= -(3^4 \cdot z^4)$
$= -81z^4$

PRACTICE

Simplify the expression.

If a and b are real numbers and m is a positive integer, $(ab)^m = a^m b^m$.

13. $(5 \cdot 4)^6$ **14.** $(3gh)^3$ **15.** $(6cd)^2$

16. $(-2p)^4$ **17.** $-(5t)^3$ **18.** $-(-8a)^2$

4. Quotient of Powers Property

Vocabulary

Quotient of Powers Property To divide powers having the same base, subtract exponents.

EXAMPLE

If a is a nonzero real number and m and n are positive integers such that $m > n$, $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$.

Simplify the expression.

a. $\frac{9^7}{9^3}$ **b.** $\frac{(-6)^{10}}{(-6)^8}$ **c.** $\frac{3^9 \cdot 3^5}{3^4}$ **d.** $\frac{1}{x^5} \cdot x^{12}$

Solution:

a. $\frac{9^7}{9^3}$
$= 9^{7-3}$
$= 9^4$

b. $\frac{(-6)^{10}}{(-6)^8}$
$= (-6)^{10-8}$
$= (-6)^2$

c. $\frac{3^9 \cdot 3^5}{3^4} = \frac{3^{14}}{3^4}$
$= 3^{14-4}$
$= 3^{10}$

d. $\frac{1}{x^5} \cdot x^{12} = \frac{x^{12}}{x^5}$
$= x^{12-5}$
$= x^7$

PRACTICE

Simplify the expression.

19. $\frac{18^{23}}{18^{17}}$ **20.** $\frac{2^{35}}{2^3}$ **21.** $\frac{(-25)^3}{(-25)}$

22. $\frac{4^6 \cdot 4^9}{4^3}$ **23.** $\frac{1}{n^8} \cdot n^{12}$ **24.** $w^6 \cdot \frac{1}{w^4}$

Name _____ Date _____

BENCHMARK 4

(Chapters 7 and 8)

5. Power of a Quotient Property

Vocabulary

Power of a Quotient Property To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

EXAMPLE

If a and b are real numbers and m is a positive integer, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$.

Simplify the expression.

a. $\left(\frac{a}{b}\right)^7$ **b.** $\left(-\frac{2}{x}\right)^3$

Solution:

a. $\left(\frac{a}{b}\right)^7 = \frac{a^7}{b^7}$

b. $\left(-\frac{2}{x}\right)^3 = \left(\frac{-2}{x}\right)^3 = \frac{(-2)^3}{x^3} = -\frac{8}{x^3}$

PRACTICE

Simplify the expression.

25. $\left(\frac{2}{5}\right)^3$ **26.** $\left(-\frac{5}{8}\right)^2$ **27.** $\left(\frac{1}{p}\right)^8$

28. $\left(\frac{r}{s}\right)^6$ **29.** $\left(-\frac{u}{v}\right)^9$ **30.** $\left(-\frac{2}{b}\right)^5$

6. Zero and Negative Exponents

Vocabulary

Zero power a to the zero power is 1.

Negative exponents a^{-n} is the reciprocal of a^n, a^n is the reciprocal of a^{-n}.

EXAMPLE

If a is a real number ($a \neq 0$) and n is an integer, $a^0 = 1$, $a^{-n} = \frac{1}{a^n}$, and $a^n = \frac{1}{a^{-n}}$.

Simplify the expression.

a. 5^{-3} **b.** $(-12)^0$ **c.** $\left(\frac{1}{2}\right)^{-5}$ **d.** 0^{-4}

Solution:

a. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

b. $(-12)^0 = 1$

c. $\left(\frac{1}{2}\right)^{-5} = \frac{1}{\left(\frac{1}{2}\right)^5} = \frac{1}{\frac{1}{32}} = 32$

d. $0^{-4} = \frac{1}{0^4}$

Division by 0 is undefined.

PRACTICE

Simplify the expression.

31. 11^{-2} **32.** $\left(\frac{5}{8}\right)^0$ **33.** $\left(\frac{1}{4}\right)^{-3}$

34. $(-3)^{-4}$ **35.** $\frac{1}{6^{-2}}$ **36.** $(-25)^0$

Name _____ Date _____

Benchmark 4

(Chapters 7 and 8)

Quiz

Simplify the expression.

1. $7^4 \cdot 7 \cdot 7^5$

2. $(-8)^3(-8)^9(-8)$

3. $y^{12} \cdot y^8 \cdot y^9$

4. $(6^5)^3$

5. $[(-4)^3]^7$

6. $(s^6)^2$

7. $[(a + 11)^8]^4$

8. $(8 \cdot 2)^{13}$

9. $(-5jk)^7$

10. $\dfrac{3^9 \cdot 3^6}{3^2}$

11. $\dfrac{1}{v^8} \cdot v^{17}$

12. $\left(-\dfrac{3}{4}\right)^3$

13. $\left(\dfrac{4}{h}\right)^2$

14. $\left(-\dfrac{p}{q}\right)^{15}$

15. $\left(\dfrac{2}{7}\right)^0$

16. $(-2)^{-5}$

17. $\dfrac{1}{7^{-2}}$

18. $(-13)^0$