5 Writing Linear Equations

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 5: evaluating functions and finding the slopes and y-intercepts of lines.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. In the equation $y = mx + b$, the value of m is the __?__ of the graph of the equation.
2. In the equation $y = mx + b$, the value of b is the __?__ of the graph of the equation.
3. Two lines are __?__ if their slopes are equal.

SKILLS CHECK

Find the slope of the line that passes through the points.
(Review p. 235 for 5.1–5.6.)

4. (4, 5), (2, 3) **5.** (0, −6), (8, 0) **6.** (0, 0), (−1, 2)

Identify the slope and the y-intercept of the line with the equation.
(Review p. 244 for 5.1–5.6.)

7. $y = x + 1$ **8.** $y = \frac{3}{4}x - 6$ **9.** $y = -\frac{2}{5}x - 2$

Evaluate the function when $x = -2$, 0, and 4. *(Review p. 262 for 5.7.)*

10. $f(x) = x - 10$ **11.** $f(x) = 2x + 4$ **12.** $f(x) = -5x - 7$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 5, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 344. You will also use the key vocabulary listed below.

Big Ideas

1. **Writing linear equations in a variety of forms**
2. **Using linear models to solve problems**
3. **Modeling data with a line of fit**

Key Vocabulary

- point-slope form, *p. 302*
- converse, *p. 319*
- perpendicular, *p. 320*
- scatter plot, *p. 325*
- correlation, *p. 325*
- line of fit, *p. 326*
- best-fitting line, *p. 335*
- linear regression, *p. 335*
- interpolation, *p. 335*
- extrapolation, *p. 336*
- zero of a function, *p. 337*

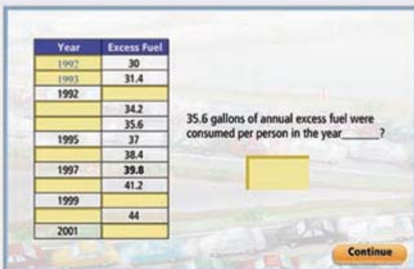
Why?

You can use linear equations to solve problems involving a constant rate of change. For example, you can write an equation that models how traffic delays affected excess fuel consumption over time.

Animated Algebra

The animation illustrated below for Exercise 40 on p. 307 helps you to answer the question: In what year was a certain amount of excess fuel consumed?

Find the year in which the given amount of excess fuel was consumed.



Click on the table in order to fill in the missing information.

Animated Algebra at classzone.com

Other animations for Chapter 5: pages 283, 303, 307, 311, 322, 327, and 335

5.1 Modeling Linear Relationships

MATERIALS • 8.5 inch by 11 inch piece of paper • inch ruler

QUESTION How can you model a linear relationship?

You know that the perimeter of a rectangle is given by the formula $P = 2\ell + 2w$. In this activity, you will find a linear relationship using that formula.

EXPLORE Find perimeters of rectangles

Width of fold (inches)	Perimeter of rectangle (inches)
0	39
1	?
2	?
3	?
4	?

STEP 1 *Find perimeter*
Find the perimeter of a piece of paper that is 8.5 inches wide and 11 inches long. Record the result in a table like the one shown.

STEP 2 *Change paper size*
Measure 1 inch from a short edge of the paper. Fold over 1 inch of the paper. You now have a rectangle with the same width and a different length than the original piece of paper. Find the perimeter of this new rectangle and record it in your table.

STEP 3 *Find additional perimeters*
Unfold the paper and repeat Step 2, this time folding the paper 2 inches from a short edge. Find the perimeter of this rectangle and record the result in your table. Repeat with a fold of 3 inches and a fold of 4 inches.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. What were the length and the width of the piece of paper before it was folded? By how much did these dimensions change with each fold?
2. What was the perimeter of the piece of paper before it was folded? By how much did the perimeter change with each fold?
3. Use the values from your table to predict the perimeter of the piece of paper after a fold of 5 inches. *Explain* your reasoning.
4. Write a rule you could use to find the perimeter of the piece of paper after a fold of n inches. Use the data in the table to show that this rule gives accurate results.

5.1 Write Linear Equations in Slope-Intercept Form

Before	You graphed equations of lines.
Now	You will write equations of lines.
Why?	So you can model distances in sports, as in Ex. 52.

Key Vocabulary
- ***y*-intercept,** *p. 225*
- **slope,** *p. 235*
- **slope-intercept form,** *p. 244*

Recall that the graph of an equation in slope-intercept form, $y = mx + b$, is a line with a slope of m and a y-intercept of b. You can use this form to write an equation of a line if you know its slope and y-intercept.

EXAMPLE 1 Use slope and *y*-intercept to write an equation

Write an equation of the line with a slope of −2 and a *y*-intercept of 5.

$y = mx + b$ **Write slope-intercept form.**

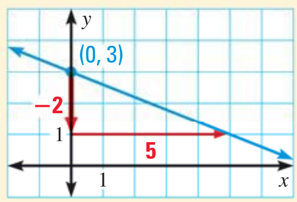
$y = -\ x + 5$ **Substitute −2 for *m* and 5 for *b*.**

EXAMPLE 2 Standardized Test Practice

Which equation represents the line shown?

Ⓐ $y = -\frac{2}{5}x + 3$ Ⓑ $y = -\frac{5}{2}x + 3$

Ⓒ $y = -\frac{2}{5}x + 1$ Ⓓ $y = 3x + \frac{2}{5}$



ELIMINATE CHOICES
In Example 2, you can eliminate choices C and D because the y-intercepts of the graphs of these equations are not 3.

The slope of the line is $\frac{\text{rise}}{\text{run}} = \frac{-2}{5} = -\frac{2}{5}$.

The line crosses the y-axis at (0, 3). So, the y-intercept is 3.

$y = mx + b$ **Write slope-intercept form.**

$y = -\frac{2}{5}x + 3$ **Substitute $-\frac{2}{5}$ for *m* and 3 for *b*.**

▸ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

Animated Algebra at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

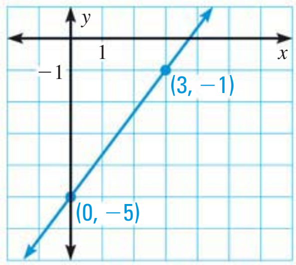
Write an equation of the line with the given slope and *y*-intercept.

1. Slope is 8; y-intercept is −7.
2. Slope is $\frac{3}{4}$; y-intercept is −3.

USING TWO POINTS If you know the point where a line crosses the y-axis and any other point on the line, you can write an equation of the line.

EXAMPLE 3 Write an equation of a line given two points

Write an equation of the line shown.



Solution

STEP 1 **Calculate** the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-5)}{3 - 0} = \frac{4}{3}$$

STEP 2 **Write** an equation of the line. The line crosses the y-axis at $(0, -5)$. So, the y-intercept is -5.

$y = mx + b$ **Write slope-intercept form.**

$y = \frac{4}{3}x - 5$ **Substitute $\frac{4}{3}$ for m and -5 for b.**

WRITING FUNCTIONS Recall that the graphs of linear functions are lines. You can use slope-intercept form to write a linear function.

EXAMPLE 4 Write a linear function

REVIEW FUNCTIONS
For help with using function notation, see p. 262.

Write an equation for the linear function f with the values $f(0) = 5$ and $f(4) = 17$.

Solution

STEP 1 **Write** $f(0) = 5$ as $(0, 5)$ and $f(4) = 17$ as $(4, 17)$.

STEP 2 **Calculate** the slope of the line that passes through $(0, 5)$ and $(4, 17)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{17 - 5}{4 - 0} = \frac{12}{4} = 3$$

STEP 3 **Write** an equation of the line. The line crosses the y-axis at $(0, 5)$. So, the y-intercept is 5.

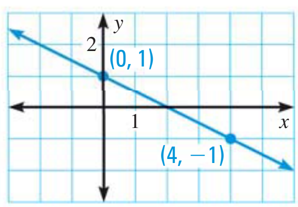
$y = mx + b$ **Write slope-intercept form.**

$y = 3x + 5$ **Substitute 3 for m and 5 for b.**

▶ The function is $f(x) = 3x + 5$.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Write an equation of the line shown.



Write an equation for the linear function f with the given values.

4. $f(0) = -2, f(8) = 4$

5. $f(-3) = 6, f(0) = 5$

READING
The value b is a starting value in a real-world situation modeled by $y = mx + b$, because when $x = 0$, the value of y is b.

MODELING REAL-WORLD SITUATIONS When a quantity y changes at a constant rate with respect to a quantity x, you can use the equation $y = mx + b$ to model the relationship. The value of m is the constant rate of change, and the value of b is an initial, or starting, value for y.

EXAMPLE 5 Solve a multi-step problem

RECORDING STUDIO A recording studio charges musicians an initial fee of \$50 to record an album. Studio time costs an additional \$35 per hour.

a. Write an equation that gives the total cost of an album as a function of studio time (in hours).

b. Find the total cost of recording an album that takes 10 hours of studio time.

Solution

a. The cost changes at a constant rate, so you can write an equation in slope-intercept form to model the total cost.

STEP 1 **Identify** the rate of change and the starting value.

Rate of change, *m*: cost per hour
Starting value, *b*: initial fee

STEP 2 **Write** a verbal model. Then write the equation.

Total cost (dollars)	=	Cost per hour (dollars per hour)	•	Studio time (hours)	+	Initial fee (dollars)
C	=	35	•	t	+	50

CHECK Use unit analysis to check the equation.

$$\text{dollars} = \frac{\text{dollars}}{\cancel{\text{hour}}} \cdot \cancel{\text{hours}} + \text{dollars} \checkmark$$

▶ The total cost C is given by the function $C = 35t + 50$ where t is the studio time (in hours).

b. Evaluate the function for $t = 10$.

$C = 35(10) + 50 = 400$ **Substitute 10 for *t* and simplify.**

▶ The total cost for 10 hours of studio time is \$400.

✓ GUIDED PRACTICE for Example 5

6. WHAT IF? In Example 5, suppose the recording studio raises its initial fee to \$75 and charges \$40 per hour for studio time.

a. Write an equation that gives the total cost of an album as a function of studio time (in hours).

b. Find the total cost of recording an album that takes 10 hours of studio time.

5.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 19, and 47

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 40, 43, 48, and 50

◆ = **MULTIPLE REPRESENTATIONS** Ex. 49

SKILL PRACTICE

1. VOCABULARY Copy and complete: The ratio of the rise to the run between any two points on a nonvertical line is called the _?_.

2. ★ WRITING *Explain* how you can use slope-intercept form to write an equation of a line given its slope and y-intercept.

EXAMPLE 1 on p. 283 for Exs. 3–9, 16

WRITING EQUATIONS **Write an equation of the line with the given slope and y-intercept.**

3. slope: 2
y-intercept: 9

4. slope: 1
y-intercept: 5

5. slope: -3
y-intercept: 0

6. slope: -7
y-intercept: 1

7. slope: $\frac{2}{3}$
y-intercept: -9

8. slope: $\frac{3}{4}$
y-intercept: -6

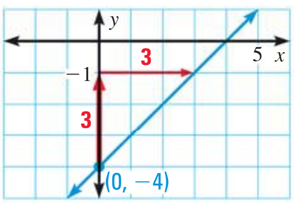
9. ★ MULTIPLE CHOICE Which equation represents the line with a slope of -1 and a y-intercept of 2?

Ⓐ $y = -x + 2$ Ⓑ $y = 2x - 1$ Ⓒ $y = x - 2$ Ⓓ $y = 2x + 1$

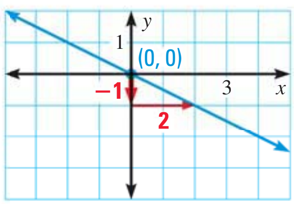
EXAMPLE 2 on p. 283 for Exs. 10–15, 17

WRITING EQUATIONS **Write an equation of the line shown.**

10.



11.



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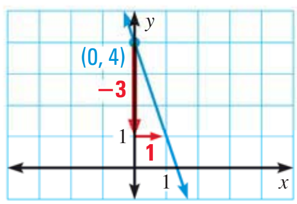

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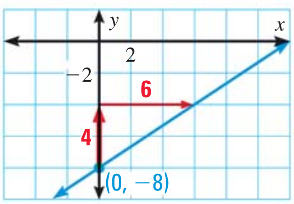

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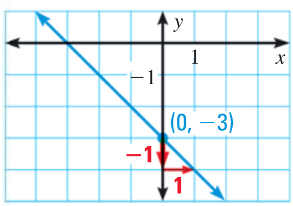
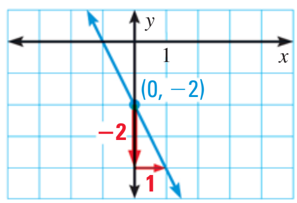


15.



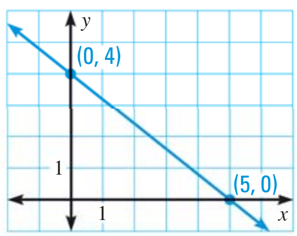
16. ERROR ANALYSIS *Describe* and correct the error in writing an equation of the line with a slope of 2 and a y-intercept of 7.

$y = 7x + 2$ ✗

17. ERROR ANALYSIS *Describe* and correct the error in writing an equation of the line shown.

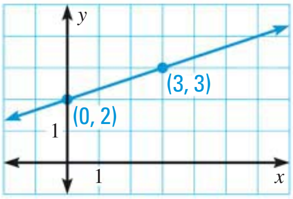
$$\text{slope} = \frac{0-4}{0-5} = \frac{-4}{-5} = \frac{4}{5}$$

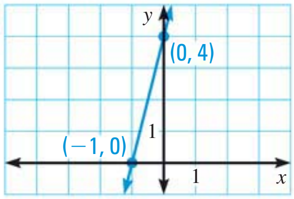
$$y = \frac{4}{5}x + 4$$ ✗

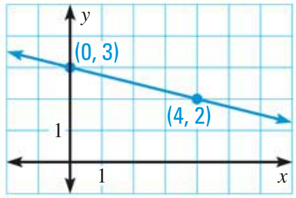


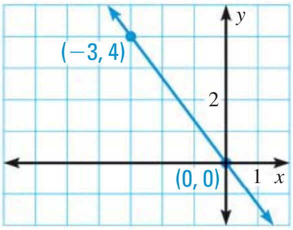
EXAMPLE 3
on p. 284
for Exs. 18–29

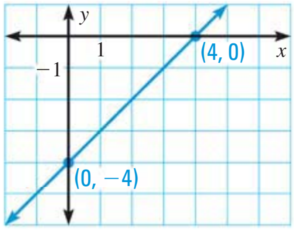
USING A GRAPH **Write an equation of the line shown.**

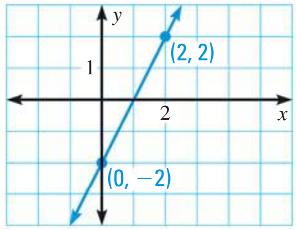
18.


19.


20.


21.


22.


23.


USING TWO POINTS **Write an equation of the line that passes through the given points.**

24. $(-3, 1), (0, -8)$

25. $(2, -7), (0, -5)$

26. $(2, -4), (0, -4)$

27. $(0, 4), (8, 3.5)$

28. $(0, 5), (1.5, 1)$

29. $(-6, 0), (0, -24)$

EXAMPLE 4
on p. 284
for Exs. 30–38

WRITING FUNCTIONS **Write an equation for the linear function *f* with the given values.**

30. $f(0) = 2, f(2) = 4$

31. $f(0) = 7, f(3) = 1$

32. $f(0) = -2, f(4) = -3$

33. $f(0) = -1, f(5) = -5$

34. $f(-2) = 6, f(0) = -4$

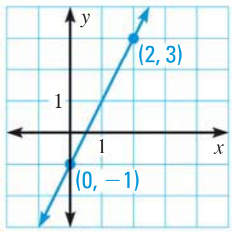
35. $f(-6) = -1, f(0) = 3$

36. $f(4) = 13, f(0) = 21$

37. $f(0) = 9, f(3) = 0$

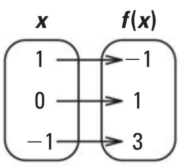
38. $f(0.2) = 1, f(0) = 0.6$

39. **VISUAL THINKING** Write an equation of the line with a slope that is half the slope of the line shown and a y-intercept that is 2 less than the y-intercept of the line shown.



40. ★ **OPEN-ENDED** *Describe* a real-world situation that can be modeled by the function $y = 4x + 9$.

USING A DIAGRAM OR TABLE **Write an equation that represents the linear function shown in the mapping diagram or table.**

41.


42.

x	$f(x)$
−4	−2
−2	−1
0	0

43. ★ **WRITING** A line passes through the points (3, 5) and (3, −7). Is it possible to write an equation of the line in slope-intercept form? *Justify* your answer.

44. **CHALLENGE** Show that the equation of the line that passes through the points $(0, b)$ and $(1, b + m)$ is $y = mx + b$. *Explain* how you can be sure that the point $(-1, b - m)$ also lies on the line.

PROBLEM SOLVING

EXAMPLE 5 on p. 285 for Exs. 45–49

45. WEB SERVER The initial fee to have a website set up using a server is \$48. It costs \$44 per month to maintain the website.

a. Write an equation that gives the total cost of setting up and maintaining a website as a function of the number of months it is maintained.

b. Find the total cost of setting up and maintaining the website for 6 months.

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46. PHOTOGRAPHS A camera shop charges \$3.99 for an enlargement of a photograph. Enlargements can be delivered for a charge of \$1.49 per order. Write an equation that gives the total cost of an order with delivery as a function of the number of enlargements. Find the total cost of ordering 8 photograph enlargements with delivery.

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47. AQUARIUM Your family spends \$30 for tickets to an aquarium and \$3 per hour for parking. Write an equation that gives the total cost of your family's visit to the aquarium as a function of the number of hours that you are there. Find the total cost of 4 hours at the aquarium.

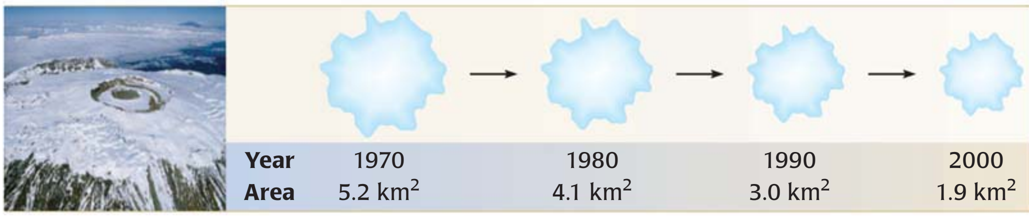
48. ★ SHORT RESPONSE Scientists found that the number of ant species in Clark Canyon, Nevada, increases at a rate of 0.0037 species per meter of elevation. There are approximately 3 ant species at sea level.

a. Write an equation that gives the number of ant species as a function of the elevation (in meters).

b. Identify the dependent and independent variables in this situation.

c. *Explain* how you can use the equation from part (a) to approximate the number of ant species at an elevation of 2 meters.

49. MULTIPLE REPRESENTATIONS The timeline shows the approximate total area of glaciers on Mount Kilimanjaro from 1970 to 2000.



a. Making a Table Make a table that shows the number of years x since 1970 and the area of the glaciers y (in square kilometers).

b. Drawing a Graph Graph the data in the table. *Explain* how you know the area of glaciers changed at a constant rate.

c. Writing an Equation Write an equation that models the area of glaciers as a function of the number of years since 1970. By how much did the area of the glaciers decrease each year from 1970 to 2000?

50. ★ **EXTENDED RESPONSE** The Harris Dam in Maine releases water into the Kennebec River. From 10:00 A.M. to 1:00 P.M. during each day of whitewater rafting season, water is released at a greater rate than usual.

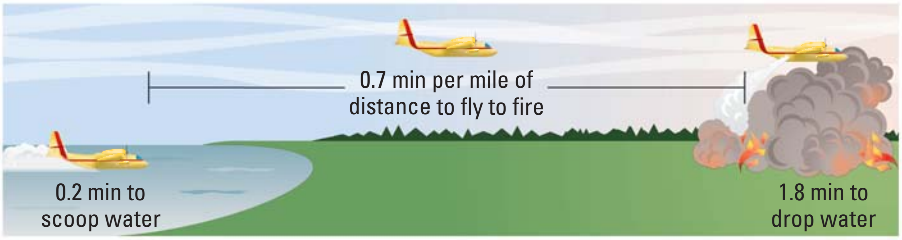
Time interval	Release rate (gallons per hour)
12:00 A.M. to 10:00 A.M.	8.1 million
10:00 A.M. to 1:00 P.M.	130 million

a. On a day during rafting season, how much water is released by 10:00 A.M.?

b. Write an equation that gives, for a day during rafting season, the total amount of water (in gallons) released as a function of the number of hours since 10:00 A.M.

c. What is the domain of the function from part (b)? *Explain.*

51. **FIREFIGHTING** The diagram shows the time a firefighting aircraft takes to scoop water from a lake, fly to a fire, and drop the water on the fire.



a. Model Write an equation that gives the total time (in minutes) that the aircraft takes to scoop, fly, and drop as a function of the distance (in miles) flown from the lake to the fire.

b. Predict Find the time the aircraft takes to scoop, fly, and drop if it travels 20 miles from the lake to the fire.

52. **CHALLENGE** The elevation at which a baseball game is played affects the distance a ball travels when hit. For every increase of 1000 feet in elevation, the ball travels about 7 feet farther. Suppose a baseball travels 400 feet when hit in a ball park at sea level.

a. Model Write an equation that gives the distance (in feet) the baseball travels as a function of the elevation of the ball park in which it is hit.

b. Justify *Justify* the equation from part (a) using unit analysis.

c. Predict If the ball were hit in exactly the same way at a park with an elevation of 3500 feet, how far would it travel?

MIXED REVIEW

Solve the equation. Check your solution.

53. $x + 11 = 6$ *(p. 134)* **54.** $x - 7 = 13$ *(p. 134)* **55.** $0.2x = -1$ *(p. 134)*

56. $3x + 9 = 21$ *(p. 141)* **57.** $2x - 3 = 25$ *(p. 141)* **58.** $4x - 8 = -10$ *(p. 141)*

PREVIEW
Prepare for Lesson 5.2 in Exs. 59–64.

Find the slope of the line that passes through the points. *(p. 235)*

59. $(-4, 6), (0, -2)$ **60.** $(-3, -2), (0, 1)$ **61.** $(5, 6), (-1, 3)$

62. $(-9, 3), (7, -1)$ **63.** $(3, -12), (5, -7)$ **64.** $(10, 4), (-8, 2)$

Graphing Calculator **ACTIVITY** *Use after Lesson 5.1*

5.1 Investigate Families of Lines

QUESTION **How can you use a graphing calculator to find equations of lines using slopes and *y*-intercepts?**

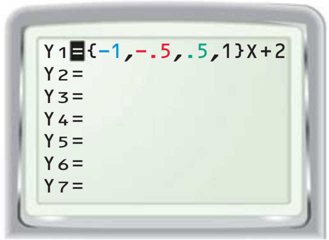
Recall from Chapter 4 that you can create families of lines by varying the value of either m or b in $y = mx + b$. The constants m and b are called *parameters*. Given the value of one parameter, you can determine the value of the other parameter if you also have information that uniquely identifies one member of the family of lines.

EXAMPLE 1 Find the slope of a line and write an equation

In the same viewing window, display the four lines that have slopes of −1, −0.5, 0.5, and 1 and a *y*-intercept of 2. Then use the graphs to determine which line passes through the point (12, 8). Write an equation of the line.

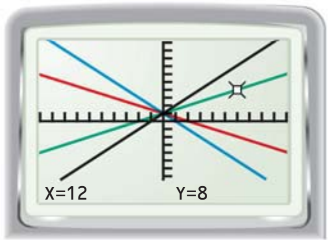
STEP 1 Enter equations

Press Y= and enter the four equations. Because the lines all have the same *y*-intercept, they constitute a family of lines and can be entered as shown.



STEP 2 Display graphs

Graph the equations in an appropriate viewing window. Press TRACE and use the left and right arrow keys to move along one of the lines until $x = 12$. Use the up and down arrow keys to see which line passes through (12, 8).



STEP 3 Find the line

The line that passes through (12, 8) is the line with a slope of 0.5. So, an equation of the line is $y = 0.5x + 2$.

PRACTICE

Display the lines that have the same *y*-intercept but different slopes, as given, in the same viewing window. Determine which line passes through the given point. Write an equation of the line.

1. Slopes: −3, −2, 2, 3; *y*-intercept: 5; point: (−3, 11)
2. Slopes: 4, −2.5, 2.5, 4; *y*-intercept: −1; point: (4, −11)
3. Slopes: −2, −1, 1, 2; *y*-intercept: 1.5; point: (1, 3.5)

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EXAMPLE 2 Find the *y*-intercept of a line and write an equation

In the same viewing window, display the five lines that have a slope of 0.5 and *y*-intercepts of −2, −1, 0, 1, and 2. Then use the graphs to determine which line passes through the point (−2, −2). Write an equation of the line.

STEP 1 Enter equations

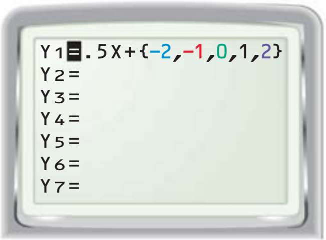
Press Y= and enter the five equations. Because the lines all have the same slope, they constitute a family of lines and can be entered as shown below.

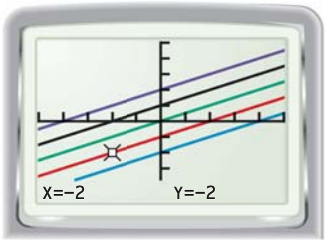
STEP 2 Display graphs

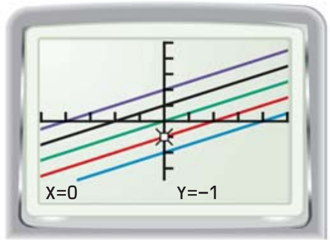
Graph the equations in an appropriate viewing window. Press TRACE and use the left and right arrow keys to move along one of the lines until $x = -2$. Use the up and down arrow keys to see which line passes through (−2, −2).

STEP 3 Find the line

The line that passes through (−2, −2) is the line with a *y*-intercept of −1. So, an equation of the line is $y = 0.5x - 1$.







PRACTICE

Display the lines that have the same slope but different *y*-intercepts, as given, in the same viewing window. Determine which line passes through the given point. Write an equation of the line.

4. Slope: −3; *y*-intercepts: −2, −1, 0, 1, 2; point: (4, −13)

5. Slope: 1.5; *y*-intercepts: −2, −1, 0, 1, 2; point: (−2, −1)

6. Slope: −0.5; *y*-intercepts: −3, −1.5, 0, 1.5, 3; point: (−4, 3.5)

7. Slope: 4; *y*-intercepts: −3, −1, 0, 1, 3; point: (2, 5)

8. Slope: 2; *y*-intercepts: −6, −3, 0, 3, 6; point: (−2, −7)

DRAW CONCLUSIONS

9. Of all the lines having equations of the form $y = 0.5x + b$, which one passes through the point (2, 2)? *Explain* how you found your answer.

10. *Describe* a process you could use to find an equation of a line that has a slope of −0.25 and passes through the point (8, −2).

5.2 Use Linear Equations in Slope-Intercept Form

Before You wrote an equation of a line using its slope and y-intercept.
Now You will write an equation of a line using points on the line.
Why So you can write a model for total cost, as in Example 5.

Key Vocabulary
- ***y*-intercept,** *p. 225*
- **slope,** *p. 235*
- **slope-intercept form,** *p. 244*

KEY CONCEPT *For Your Notebook*

Writing an Equation of a Line in Slope-Intercept Form

STEP 1 **Identify** the slope m. You can use the slope formula to calculate the slope if you know two points on the line.

STEP 2 **Find** the y-intercept. You can substitute the slope and the coordinates of a point (x, y) on the line in $y = mx + b$. Then solve for b.

STEP 3 **Write** an equation using $y = mx + b$.

EXAMPLE 1 Write an equation given the slope and a point

Write an equation of the line that passes through the point $(-1, 3)$ and has a slope of -4.

Solution

STEP 1 **Identify** the slope. The slope is -4.

STEP 2 **Find** the y-intercept. Substitute the slope and the coordinates of the given point in $y = mx + b$. Solve for b.

$y = mx + b$ Write slope-intercept form.

$3 = -4(-1) + b$ Substitute -4 for m, -1 for x, and 3 for y.

$-1 = b$ Solve for b.

AVOID ERRORS
When you substitute, be careful not to mix up the x- and y-values.

STEP 3 **Write** an equation of the line.

$y = mx + b$ Write slope-intercept form.

$y = -4x - 1$ Substitute -4 for m and -1 for b.

✓ GUIDED PRACTICE for Example 1

1. Write an equation of the line that passes through the point $(6, 3)$ and has a slope of 2.

EXAMPLE 2 Write an equation given two points

Write an equation of the line that passes through (−2, 5) and (2, −1).

Solution

STEP 1 **Calculate** the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{2 - (-2)} = \frac{-6}{4} = -\frac{3}{2}$$

ANOTHER WAY

You can also find the y-intercept using the coordinates of the other given point, (2, −1):

$y = mx + b$

$-1 = -\frac{3}{2}(2) + b$

$2 = b$

STEP 2 **Find** the y-intercept. Use the slope and the point (−2, 5).

$y = mx + b$ — Write slope-intercept form.

$5 = -\frac{3}{2}(-2) + b$ — Substitute $-\frac{3}{2}$ for m, −2 for x, and 5 for y.

$2 = b$ — Solve for b.

STEP 3 **Write** an equation of the line.

$y = mx + b$ — Write slope-intercept form.

$y = -\frac{3}{2}x + 2$ — Substitute $-\frac{3}{2}$ for m and 2 for b.

EXAMPLE 3 Standardized Test Practice

Which function has the values $f(4) = 9$ and $f(-4) = -7$?

Ⓐ $f(x) = 2x + 10$ Ⓑ $f(x) = 2x + 1$

Ⓒ $f(x) = 2x - 13$ Ⓓ $f(x) = 2x - 14$

ELIMINATE CHOICES

You can also evaluate each function when $x = 4$ and $x = -4$. Eliminate any choices for which $f(4) \neq 9$ or $f(-4) \neq -7$.

STEP 1 **Calculate** the slope. Write $f(4) = 9$ as (4, 9) and $f(-4) = -7$ as (−4, −7).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 9}{-4 - 4} = \frac{-16}{-8} = 2$$

STEP 2 **Find** the y-intercept. Use the slope and the point (4, 9).

$y = mx + b$ — Write slope-intercept form.

$9 = 2(4) + b$ — Substitute 2 for m, 4 for x, and 9 for y.

$1 = b$ — Solve for b.

STEP 3 **Write** an equation for the function. Use function notation.

$f(x) = 2x + 1$ — Substitute 2 for m and 1 for b.

▶ The answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 2 and 3

2. Write an equation of the line that passes through (1, −2) and (−5, 4).

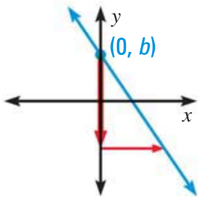
3. Write an equation for the linear function with the values $f(-2) = 10$ and $f(4) = -2$.

CONCEPT SUMMARY

For Your Notebook

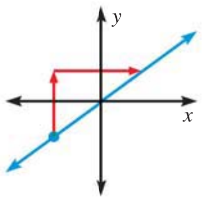
How to Write Equations in Slope-Intercept Form

Given slope m and y-intercept b



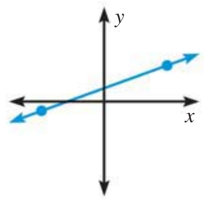
Substitute m and b in the equation $y = mx + b$.

Given slope m and one point



Substitute m and the coordinates of the point in $y = mx + b$. Solve for b. Write the equation.

Given two points



Use the points to find the slope m. Then follow the same steps described at the left.

MODELING REAL-WORLD SITUATIONS You can model a real-world situation that involves a constant rate of change with an equation in slope-intercept form.

EXAMPLE 4 Solve a multi-step problem

GYM MEMBERSHIP Your gym membership costs \$33 per month after an initial membership fee. You paid a total of \$228 after 6 months. Write an equation that gives the total cost as a function of the length of your gym membership (in months). Find the total cost after 9 months.

Solution

STEP 1 **Identify** the rate of change and starting value.

Rate of change, m: monthly cost, \$33 per month
Starting value, b: initial membership fee

STEP 2 **Write** a verbal model. Then write an equation.

Total cost	=	Monthly cost	•	Number of months	+	Membership fee
↓		↓		↓		↓
C	=	33	•	t	+	b

STEP 3 **Find** the starting value. Membership for 6 months costs \$228, so you can substitute 6 for t and 228 for C in the equation $C = 33t + b$.

$228 = 33(6) + b$ **Substitute 6 for t and 228 for C.**

$30 = b$ **Solve for b.**

STEP 4 **Write** an equation. Use the function from Step 2.

$C = 33t + 30$ **Substitute 30 for b.**

STEP 5 **Evaluate** the function when $t = 9$.

$C = 33(9) + 30 = 327$ **Substitute 9 for t. Simplify.**

▶ Your total cost after 9 months is \$327.

EXAMPLE 5 Solve a multi-step problem

BMX RACING In Bicycle Moto Cross (BMX) racing, racers purchase a one year membership to a track. They also pay an entry fee for each race at that track. One racer paid a total of \$125 after 5 races. A second racer paid a total of \$170 after 8 races. How much does the track membership cost? What is the entry fee per race?

ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 300 for the **Problem Solving Workshop**.

Solution

STEP 1 **Identify** the rate of change and starting value.

Rate of change, *m*: entry fee per race
Starting value, *b*: track membership cost

STEP 2 **Write** a verbal model. Then write an equation.

Total cost	=	Entry fee per race	•	Races entered	+	Membership cost
C	=	m	•	r	+	b

STEP 3 **Calculate** the rate of change. This is the entry fee per race. Use the slope formula. Racer 1 is represented by (5, 125). Racer 2 is represented by (8, 170).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{170 - 125}{8 - 5} = \frac{45}{3} = 15$$

STEP 4 **Find** the track membership cost b. Use the data pair (5, 125) for racer 1 and the entry fee per race from Step 3.

$C = mr + b$	Write the equation from Step 2.
$125 = 15(5) + b$	Substitute 15 for *m*, 5 for *r*, and 125 for *C*.
$50 = b$	Solve for *b*.

▶ The track membership cost is \$50. The entry fee per race is \$15.

✓ GUIDED PRACTICE for Examples 4 and 5

4. **GYM MEMBERSHIP** A gym charges \$35 per month after an initial membership fee. A member has paid a total of \$250 after 6 months. Write an equation that gives the total cost of a gym membership as a function of the length of membership (in months). Find the total cost of membership after 10 months.

5. **BMX RACING** A BMX race track charges a membership fee and an entry fee per race. One racer paid a total of \$76 after 3 races. Another racer paid a total of \$124 after 7 races.

   a. How much does the track membership cost?

   b. What is the entry fee per race?

   c. Write an equation that gives the total cost as a function of the number of races entered.

5.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 11, and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 29, 34–37, 41, and 49

◆ = **MULTIPLE REPRESENTATIONS** Ex. 53

SKILL PRACTICE

1. **VOCABULARY** What is the y-coordinate of a point where a graph crosses the y-axis called?

2. ★ **WRITING** If the equation $y = mx + b$ is used to model a quantity y as a function of the quantity x, why is b considered to be the starting value?

EXAMPLE 1 on p. 292 for Exs. 3–9

WRITING EQUATIONS Write an equation of the line that passes through the given point and has the given slope m.

3. $(1, 1); m = 3$
4. $(5, 1); m = 2$
5. $(-4, 7); m = -5$
6. $(5, -5); m = -2$
7. $(8, -4); m = -\frac{3}{4}$
8. $(-3, -11); m = \frac{1}{2}$

9. **ERROR ANALYSIS** *Describe* and correct the error in finding the y-intercept of the line that passes through the point $(6, -3)$ and has a slope of -2.

$y = mx + b$
$6 = -2(-3) + b$
$6 = 6 + b$
$0 = b$

EXAMPLE 4 on p. 294 for Ex. 10

10. **ERROR ANALYSIS** An Internet service provider charges $18 per month plus an initial set-up fee. One customer paid a total of $81 after 2 months of service. *Describe* and correct the error in finding the set-up fee.

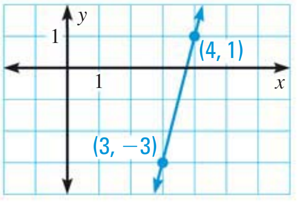
$C = mt + b$
$81 = m(2) + 18$
$63 = m(2)$
$31.50 = m$

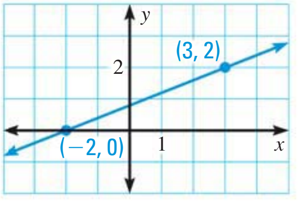
EXAMPLE 2 on p. 293 for Exs. 11–22

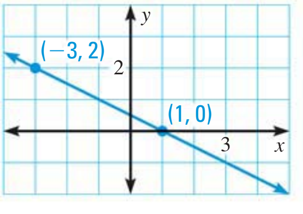
USING TWO POINTS Write an equation of the line that passes through the given points.

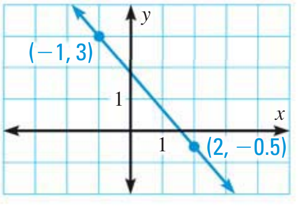
11. $(1, 4), (2, 7)$
12. $(3, 2), (4, 9)$
13. $(10, -5), (-5, 1)$
14. $(-2, 8), (-6, 0)$
15. $\left(\frac{9}{2}, 1\right), \left(-\frac{7}{2}, 7\right)$
16. $\left(-5, \frac{3}{4}\right), \left(-2, -\frac{3}{4}\right)$

USING A GRAPH Write an equation of the line shown.

17.


18.


19.


20.


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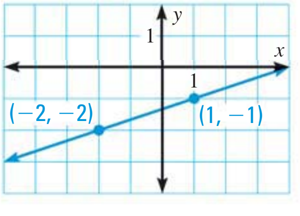

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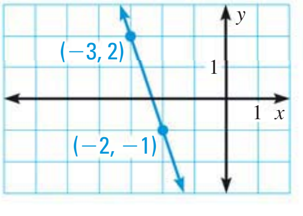


EXAMPLE 3
on p. 293
for Exs. 23–33

WRITING LINEAR FUNCTIONS **Write an equation for a linear function f that has the given values.**

23. $f(-2) = 15, f(1) = 9$

24. $f(-2) = -2, f(4) = -8$

25. $f(2) = 7, f(4) = 6$

26. $f(-4) = -8, f(-8) = -11$

27. $f(3) = 1, f(6) = 4$

28. $f(-5) = 9, f(11) = -39$

29. ★ **MULTIPLE CHOICE** Which function has the values $f(4) = -15$ and $f(7) = 57$?

(A) $f(x) = 14x - 71$ (B) $f(x) = 24x - 1361$

(C) $f(x) = 24x + 360$ (D) $f(x) = 24x - 111$

USING A TABLE OR DIAGRAM **Write an equation that represents the linear function shown in the table or mapping diagram.**

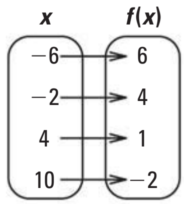
30.

x	$f(x)$
−4	6
4	4
8	3
12	2

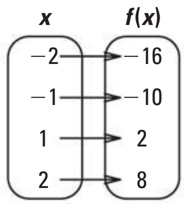
31.

x	$f(x)$
−3	8
3	4
6	2
9	0

32.



33.



★ **SHORT RESPONSE** **Tell whether the given information is enough to write an equation of a line. *Justify* your answer.**

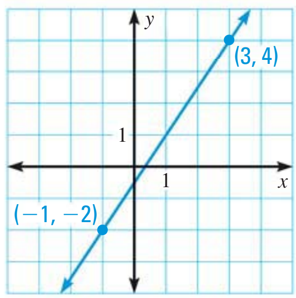
34. Two points on the line

35. The slope and a point on the line

36. The slope of the line

37. Both intercepts of the line

USING A GRAPH **In Exercises 38–41, use the graph at the right.**



38. Write an equation of the line shown.

39. Write an equation of a line that has the same y-intercept as the line shown but has a slope that is 3 times the slope of the line shown.

40. Write an equation of a line that has the same slope as the line shown but has a y-intercept that is 6 more than the y-intercept of the line shown.

41. ★ **WRITING** Which of the lines from Exercises 38–40 intersect? Which of the lines never intersect? *Justify* your answers.

REASONING **Decide whether the three points lie on the same line. *Explain* how you know. If the points do lie on the same line, write an equation of the line that passes through all three points.**

42. $(-4, -2), (2, 2.5), (8, 7)$

43. $(2, 2), (-4, 5), (6, 1)$

44. $(-10, 4), (-3, 2.8), (-17, 6.8)$

45. $(-5.5, 3), (-7.5, 4), (-4, 5)$

46. **CHALLENGE** A line passes through the points $(-2, 3)$, $(2, 5)$, and $(6, k)$. Find the value of k. *Explain* your steps.

PROBLEM SOLVING

EXAMPLES 4 and 5
on pp. 294–295 for Exs. 47–50

47. BIOLOGY Four years after a hedge maple tree was planted, its height was 9 feet. Eight years after it was planted, the hedge maple tree's height was 12 feet. What is the growth rate of the hedge maple? What was its height when it was planted?

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48. TECHNOLOGY You have a subscription to an online magazine that allows you to view 25 articles from the magazine's archives. You are charged an additional fee for each article after the first 25 articles viewed. After viewing 28 archived articles, you paid a total of \$34.80. After viewing 30 archived articles, you paid a total of \$40.70.

a. What is the cost per archived article after the first 25 articles viewed?

b. What is cost of the magazine subscription?

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49. ★ **SHORT RESPONSE** You are cooking a roast beef until it is well-done. You must allow 30 minutes of cooking time for every pound of beef, plus some extra time. The last time you cooked a 2 pound roast, it was well-done after 1 hour and 25 minutes. How much time will it take to cook a 3 pound roast? *Explain* how you found your answer.

HINT
In part (b), let *t* represent the number of years since 1981.

50. TELEPHONE SERVICE The annual household cost of telephone service in the United States increased at a relatively constant rate of \$27.80 per year from 1981 to 2001. In 2001 the annual household cost of telephone service was \$914.

a. What was the annual household cost of telephone service in 1981?

b. Write an equation that gives the annual household cost of telephone service as a function of the number of years since 1981.

c. Find the household cost of telephone service in 2000.

51. NEWSPAPERS Use the information in the article about the circulation of Sunday newspapers.

a. About how many Sunday newspapers were in circulation in 1970?

b. Write an equation that gives the number of Sunday newspapers in circulation as a function of the number of years since 1970.

c. About how many Sunday newspapers were in circulation in 2000?

> Sunday Edition **C9**
>
> **SUNDAY PAPERS INCREASE**
>
> **From 1970 to 2000,** the number of Sunday newspapers in circulation increased at a relatively constant rate of 11.8 newspapers per year. In 1997 there were 903 Sunday newspapers in circulation.

52. AIRPORTS From 1990 to 2001, the number of airports in the United States increased at a relatively constant rate of 175 airports per year. There were 19,306 airports in the United States in 2001.

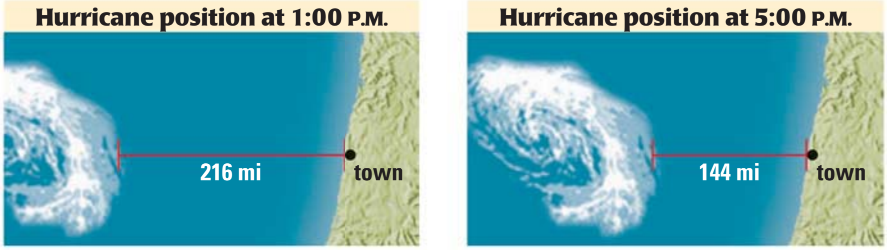
a. How many U.S. airports were there in 1990?

b. Write an equation that gives the number of U.S. airports as a function of the number of years since 1990.

c. Find the year in which the number of U.S. airports reached 19,500.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

53. **MULTIPLE REPRESENTATIONS** A hurricane is traveling at a constant speed on a straight path toward a coastal town, as shown below.



a. Writing an Equation Write an equation that gives the distance (in miles) of the hurricane from the town as a function of the number of hours since 12:00 P.M.

b. Drawing a Graph Graph the equation from part (a). *Explain* what the slope and the y-intercept of the graph mean in this situation.

c. Describing in Words Predict the time at which the hurricane will reach the town. Your answer should include the following information:

- an explanation of how you used your equation
- a description of the steps you followed to obtain your prediction

54. **CHALLENGE** An in-line skater practices at a race track. In two trials, the skater travels the same distance going from a standstill to his top racing speed. He then travels at his top racing speed for different distances.

Trial number	Time at top racing speed (seconds)	Total distance traveled (meters)
1	24	300
2	29	350

a. Model Write an equation that gives the total distance traveled (in meters) as a function of the time (in seconds) at top racing speed.

b. Justify What do the rate of change and initial value in your equation represent? *Explain* your answer using unit analysis.

c. Predict One lap around the race track is 200 meters. The skater starts at a standstill and completes 3 laps. Predict the number of seconds the skater travels at his top racing speed. *Explain* your method.

MIXED REVIEW

Solve the equation. Check your solution.

55. $3x + 2x - 3 = 12$ *(p. 148)*

56. $-2(q + 13) - 8 = 2$ *(p. 148)*

57. $-3a + 15 = 45 + 7a$ *(p. 154)*

58. $7c + 25 = -19 + 2c$ *(p. 154)*

PREVIEW
Prepare for Lesson 5.3 in Exs. 59–64.

Write an equation of the line that has the given characteristics. *(p. 283)*

59. Slope: -5; y-intercept: -2

60. Slope: $\frac{2}{7}$; y-intercept: -3

61. Slope: 1; passes through $(0, -4)$

62. Slope: 9; passes through $(0, 14)$

63. Passes through $(0, 6)$, $(5, 2)$

64. Passes through $(-12, 3)$, $(0, 2)$

Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 295

MULTIPLE REPRESENTATIONS In Example 5 on page 295, you saw how to solve a problem about BMX racing using an equation. You can also solve this problem using a graph or a table.

PROBLEM

BMX RACING In Bicycle Moto Cross (BMX) racing, racers purchase a one year membership to a track. They also pay an entry fee for each race at that track. One racer paid a total of $125 after 5 races. A second racer paid a total of $170 after 8 races. How much does the track membership cost? What is the entry fee per race?

METHOD 1

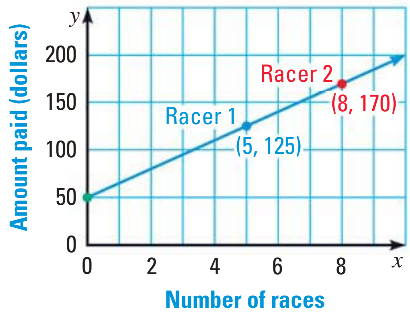
Using a Graph One alternative approach is to use a graph.

STEP 1 **Read** the problem. It tells you the number of races and amount paid for each racer. Write this information as ordered pairs.

Racer 1: (5, 125)
Racer 2: (8, 170)

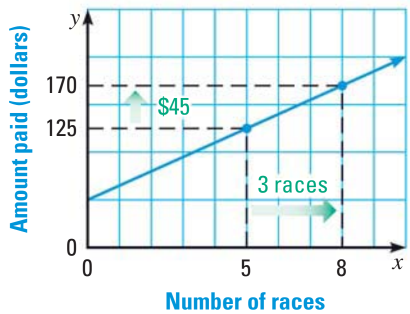
STEP 2 **Graph** the ordered pairs. Draw a line through the points.

The y-intercept is 50.
So, the track membership is $50.



STEP 3 **Find** the slope of the line. This is the entry fee per race.

$$\text{Fee} = \frac{45 \text{ dollars}}{3 \text{ races}} = \$15 \text{ per race}$$



METHOD 2 **Using a Table** Another approach is to use a table showing the amount paid for various numbers of races.

STEP 1 **Calculate** the race entry fee.

Number of races	Amount paid
5	$125
6	?
7	?
8	$170

+ 3 + $45

The number of races increased by 3, and the amount paid increased by $45, so the race entry fee is $\$45 \div 3 = \$15$.

STEP 2 **Find** the membership cost.

Number of races	Amount paid
0	$50
1	$65
2	$80
3	$95
4	$110
5	$125

− $15
− $15
− $15
− $15
− $15

The membership cost is the cost with no races. Use the race entry fee and work backwards to fill in the table. The membership cost is $50.

PRACTICE

1. **CALENDARS** A company makes calendars from personal photos. You pay a delivery fee for each order plus a cost per calendar. The cost of 2 calendars plus delivery is $43. The cost of 4 calendars plus delivery is $81. What is the delivery fee? What is the cost per calendar? Solve this problem using two different methods.

2. **BOOKSHELVES** A furniture maker offers bookshelves that have the same width and depth but that differ in height and price, as shown in the table. Find the cost of a bookshelf that is 72 inches high. Solve this problem using two different methods.

Height (inches)	Price (dollars)
36	56.54
48	77.42
60	98.30

3. **WHAT IF?** In Exercise 2, suppose the price of the 60 inch bookshelf was $99.30. Can you still solve the problem? *Explain*.

4. **CONCERT TICKETS** All tickets for a concert are the same price. The ticket agency adds a fixed fee to every order. A person who orders 5 tickets pays $93. A person who orders 3 tickets pays $57. How much will 4 tickets cost? Solve this problem using two different methods.

5. **ERROR ANALYSIS** A student solved the problem in Exercise 4 as shown below. *Describe* and correct the error.

Let p = price paid for 4 tickets

$$\frac{57}{3} = \frac{p}{4}$$

$$228 = 3p$$

$$76 = p$$

5.3 Write Linear Equations in Point-Slope Form

Before You wrote linear equations in slope-intercept form.

Now You will write linear equations in point-slope form.

Why? So you can model sports statistics, as in Ex. 43.

Key Vocabulary
- **point-slope form**

Consider the line that passes through the point (2, 3) with a slope of $\frac{1}{2}$.

Let (x, y) where $x \neq 2$ be another point on the line. You can write an equation relating x and y using the slope formula, with $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (x, y)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Write slope formula.}$$

$$\frac{1}{2} = \frac{y - 3}{x - 2} \qquad \text{Substitute } \tfrac{1}{2} \text{ for } m\text{, 3 for } y_1\text{, and 2 for } x_1.$$

$$\frac{1}{2}(x - 2) = y - 3 \qquad \text{Multiply each side by } (x - 2).$$

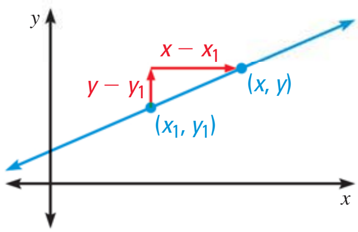
USE POINT-SLOPE FORM
When an equation is in point-slope form, you can read the x- and y-coordinates of a point on the line and the slope of the line.

The equation in *point-slope form* is $y - 3 = \frac{1}{2}(x - 2)$.

KEY CONCEPT *For Your Notebook*

Point-Slope Form

The **point-slope form** of the equation of the nonvertical line through a given point (x_1, y_1) with a slope of m is $y - y_1 = m(x - x_1)$.



EXAMPLE 1 Write an equation in point-slope form

Write an equation in point-slope form of the line that passes through the point (4, −3) and has a slope of 2.

$$y - y_1 = m(x - x_1) \qquad \text{Write point-slope form.}$$

$$y + 3 = 2(x - 4) \qquad \text{Substitute 2 for } m\text{, 4 for } x_1\text{, and } -3 \text{ for } y_1.$$

✓ GUIDED PRACTICE for Example 1

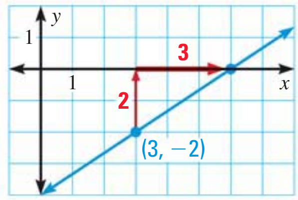
1. Write an equation in point-slope form of the line that passes through the point (−1, 4) and has a slope of −2.

EXAMPLE 2 Graph an equation in point-slope form

Graph the equation $y + 2 = \frac{2}{3}(x - 3)$.

Solution

Because the equation is in point-slope form, you know that the line has a slope of $\frac{2}{3}$ and passes through the point $(3, -2)$.



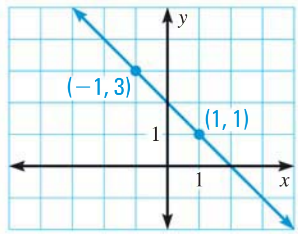
Plot the point $(3, -2)$. Find a second point on the line using the slope. Draw a line through both points.

✓ GUIDED PRACTICE for Example 2

2. Graph the equation $y - 1 = -(x - 2)$.

EXAMPLE 3 Use point-slope form to write an equation

Write an equation in point-slope form of the line shown.



Solution

STEP 1 **Find** the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{-1 - 1} = \frac{2}{-2} = \mathbf{-1}$$

STEP 2 **Write** the equation in point-slope form. You can use either given point.

Method 1 Use $(-1, 3)$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -(x + 1)$$

Method 2 Use $(1, 1)$.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -(x - 1)$$

CHECK Check that the equations are equivalent by writing them in slope-intercept form.

$$y - 3 = -x - 1 \qquad y - 1 = -x + 1$$
$$y = -x + 2 \qquad y = -x + 2$$

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✓ GUIDED PRACTICE for Example 3

3. Write an equation in point-slope form of the line that passes through the points $(2, 3)$ and $(4, 4)$.

EXAMPLE 4 Solve a multi-step problem

STICKERS You are designing a sticker to advertise your band. A company charges \$225 for the first 1000 stickers and \$80 for each additional 1000 stickers. Write an equation that gives the total cost (in dollars) of stickers as a function of the number (in thousands) of stickers ordered. Find the cost of 9000 stickers.

Solution

STEP 1 **Identify** the rate of change and a data pair. Let C be the cost (in dollars) and s be the number of stickers (in thousands).

Rate of change, m: \$80 per 1 thousand stickers
Data pair (s_1, C_1): (1 thousand stickers, \$225)

STEP 2 **Write** an equation using point-slope form. Rewrite the equation in slope-intercept form so that cost is a function of the number of stickers.

$C - C_1 = m(s - s_1)$ **Write point-slope form.**

$C - 225 = 80(s - 1)$ **Substitute 80 for m, 1 for s_1, and 225 for C_1.**

$C = 80s + 145$ **Solve for C.**

AVOID ERRORS
Remember that s is given in thousands. To find the cost of 9000 stickers, substitute 9 for s.

STEP 3 **Find** the cost of 9000 stickers.

$C = 80(9) + 145 = 865$ **Substitute 9 for s. Simplify.**

▶ The cost of 9000 stickers is \$865.

EXAMPLE 5 Write a real-world linear model from a table

WORKING RANCH The table shows the cost of visiting a working ranch for one day and night for different numbers of people. Can the situation be modeled by a linear equation? *Explain*. If possible, write an equation that gives the cost as a function of the number of people in the group.

Number of people	4	6	8	10	12
Cost (dollars)	250	350	450	550	650

Solution

STEP 1 **Find** the rate of change for consecutive data pairs in the table.

$$\frac{350 - 250}{6 - 4} = 50, \quad \frac{450 - 350}{8 - 6} = 50, \quad \frac{550 - 450}{10 - 8} = 50, \quad \frac{650 - 550}{12 - 10} = 50$$

Because the cost increases at a constant rate of \$50 per person, the situation can be modeled by a linear equation.

STEP 2 **Use** point-slope form to write the equation. Let C be the cost (in dollars) and p be the number of people. Use the data pair (4, 250).

$C - C_1 = m(p - p_1)$ **Write point-slope form.**

$C - 250 = 50(p - 4)$ **Substitute 50 for m, 4 for p_1, and 250 for C_1.**

$C = 50p + 50$ **Solve for C.**

✓ GUIDED PRACTICE for Examples 4 and 5

4. **WHAT IF?** In Example 4, suppose a second company charges \$250 for the first 1000 stickers. The cost of each additional 1000 stickers is \$60.

   a. Write an equation that gives the total cost (in dollars)of the stickers as a function of the number (in thousands) of stickers ordered.

   b. Which company would charge you less for 9000 stickers?

5. **MAILING COSTS** The table shows the cost (in dollars) of sending a single piece of first class mail for different weights. Can the situation be modeled by a linear equation? *Explain*. If possible, write an equation that gives the cost of sending a piece of mail as a function of its weight (in ounces).

Weight (ounces)	1	4	5	10	12
Cost (dollars)	0.37	1.06	1.29	2.44	2.90

5.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3 and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 30–34, 38, and 41

SKILL PRACTICE

1. **VOCABULARY** Identify the slope of the line given by the equation $y - 5 = -2(x + 5)$. Then identify one point on the line.

2. ★ **WRITING** *Describe* the steps you would take to write an equation in point-slope form of the line that passes through the points $(3, -2)$ and $(4, 5)$.

EXAMPLE 1 on p. 302 for Exs. 3–13

WRITING EQUATIONS **Write an equation in point-slope form of the line that passes through the given point and has the given slope m.**

3. $(2, 1), m = 2$
4. $(3, 5), m = -1$
5. $(7, -1), m = -6$
6. $(5, -1), m = -2$
7. $(-8, 2), m = 5$
8. $(-6, 6), m = \frac{3}{2}$
9. $(-11, -3), m = -9$
10. $(-3, -9), m = \frac{7}{3}$
11. $(5, -12), m = -\frac{2}{5}$

12. ★ **MULTIPLE CHOICE** Which equation represents the line that passes through the point $(-6, 2)$ and has a slope of -1?

    Ⓐ $y + 2 = -(x + 6)$ Ⓑ $y + 2 = -(x - 6)$

    Ⓒ $y - 2 = -(x + 6)$ Ⓓ $y + 1 = -2(x + 6)$

13. **ERROR ANALYSIS** *Describe* and correct the error in writing an equation of the line that passes through the point $(1, -5)$ and has a slope of -2.

$$y - 5 = -2(x - 1)$$

EXAMPLE 2
on p. 303
for Exs. 14–19

GRAPHING EQUATIONS Graph the equation.

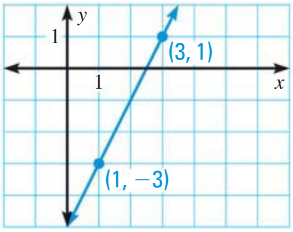
14. $y - 5 = 3(x - 1)$ **15.** $y + 3 = -2(x - 2)$ **16.** $y - 1 = 3(x + 6)$

17. $y + 8 = -(x + 4)$ **18.** $y - 1 = \frac{3}{4}(x + 1)$ **19.** $y + 4 = -\frac{5}{2}(x - 3)$

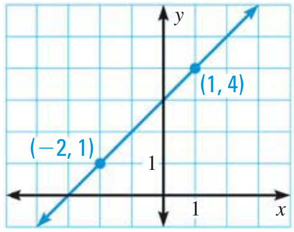
EXAMPLE 3
on p. 303
for Exs. 20–30

USING A GRAPH Write an equation in point-slope form of the line shown.

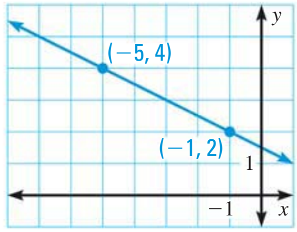
20.



21.



22.



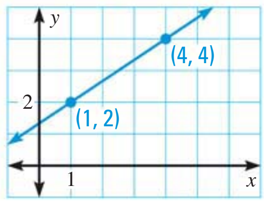
WRITING EQUATIONS Write an equation in point-slope form of the line that passes through the given points.

23. (7, 2), (2, 12) **24.** (6, −2), (12, 1) **25.** (−4, −1), (6, −7)

26. (4, 5), (−4, −5) **27.** (−3, −20), (4, 36) **28.** (−5, −19), (5, 13)

29. ERROR ANALYSIS *Describe* and correct the error in writing an equation of the line shown.

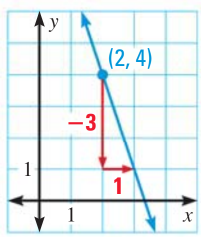
$m = \frac{4 - 2}{4 - 1} = \frac{2}{3}$ $y - 2 = \frac{2}{3}(x - 4)$



30. ★ **MULTIPLE CHOICE** The graph of which equation is shown?

Ⓐ $y + 4 = -3(x + 2)$ Ⓑ $y - 4 = -3(x - 2)$

Ⓒ $y - 4 = -3(x + 2)$ Ⓓ $y + 4 = -3(x + 2)$



★ **SHORT RESPONSE** Tell whether the data in the table can be modeled by a linear equation. *Explain*. If possible, write an equation in point-slope form that relates *y* and *x*.

31.

x	2	4	6	8	10
y	−1	5	15	29	47

32.

x	1	2	3	5	7
y	1.2	1.4	1.6	2	2.4

33.

x	1	2	3	4	5
y	2	−3	4	−5	6

34.

x	−3	−1	1	3	5
y	16	10	4	−2	−8

CHALLENGE Find the value of *k* so that the line passing through the given points has slope *m*. Write an equation of the line in point-slope form.

35. $(k, 4k), (k + 2, 3k), m = -1$ **36.** $(-k + 1, 3), (3, k + 3), m = 3$

PROBLEM SOLVING

EXAMPLE 4
on p. 304
for Exs. 37, 39, 40

37. **TELEVISION** In order to use an excerpt from a movie in a new television show, the television producer must pay the director of the movie $790 for the first 2 minutes of the excerpt and $130 per minute after that.

a. Write an equation that gives the total cost (in dollars) of using the excerpt as a function of the length (in minutes) of the excerpt.

b. Find the total cost of using an excerpt that is 8 minutes long.

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EXAMPLE 5
on p. 304
for Exs. 38, 40

38. ★ **SHORT RESPONSE** A school district pays an installation fee and a monthly fee for Internet service. The table shows the total cost of Internet service for the school district over different numbers of months. *Explain* why the situation can be modeled by a linear equation. What is the installation fee? What is the monthly service fee?

Months of service	2	4	6	8	10	12
Total cost (dollars)	9,378	12,806	16,234	19,662	23,090	26,518

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39. **COMPANY SALES** During the period 1994–2004, the annual sales of a small company increased by $10,000 per year. In 1997 the annual sales were $97,000. Write an equation that gives the annual sales as a function of the number of years since 1994. Find the sales in 2000.

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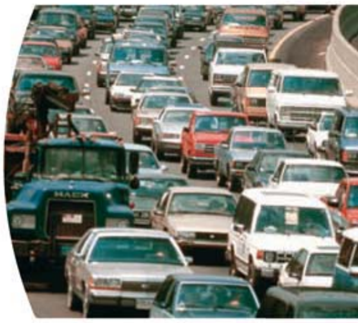
40. **TRAFFIC DELAYS** From 1990 to 2001 in Boston, Massachusetts, the annual excess fuel (in gallons per person) consumed due to traffic delays increased by about 1.4 gallons per person each year. In 1995 each person consumed about 37 gallons of excess fuel.



a. Write an equation that gives the annual excess fuel (in gallons per person) as a function of the number of years since 1990.

b. How much excess fuel was consumed per person in 2001?

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41. ★ **EXTENDED RESPONSE** The table shows the cost of ordering sets of prints of digital photos from an online service. The cost per print is the same for the first 30 prints. There is also a shipping charge.

Number of prints	1	2	5	8
Total cost (dollars)	1.98	2.47	3.94	5.41

a. *Explain* why the situation can be modeled by a linear equation.

b. Write an equation in point-slope form that relates the total cost (in dollars) of a set of prints to the number of prints ordered.

c. Find the shipping charge for up to 10 prints.

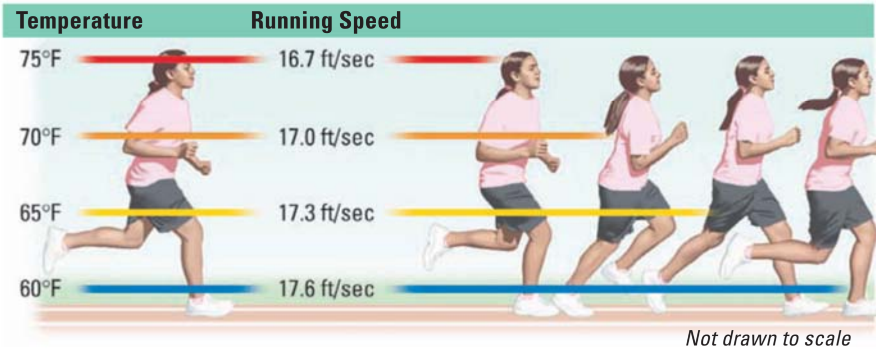
d. The cost of 15 prints is $9.14. The shipping charge increases after the first 10 prints. Find the shipping charge for 15 prints.

42. **AQUACULTURE** Aquaculture is the farming of fish and other aquatic animals. World aquaculture increased at a relatively constant rate from 1991 to 2002. In 1994 world aquaculture was about 20.8 million metric tons. In 2000 world aquaculture was about 35.5 million metric tons.

a. Write an equation that gives world aquaculture (in millions of metric tons) as a function of the number of years since 1991.

b. In 2001 China was responsible for 70.2% of world aquaculture. Approximate China's aquaculture in 2001.

43. **MARATHON** The diagram shows a marathon runner's speed at several outdoor temperatures.



a. Write an equation in point-slope form that relates running speed (in feet per second) to temperature (in degrees Fahrenheit).

b. Estimate the runner's speed when the temperature is 80°F.

44. **CHALLENGE** The number of cans recycled per pound of aluminum recycled in the U.S. increased at a relatively constant rate from 1972 to 2002. In 1977 about 23.5 cans per pound of aluminum were recycled. In 2000, about 33.1 cans per pound of aluminum were recycled.

a. Write an equation that gives the number of cans recycled per pound of aluminum recycled as a function of the number of years since 1972.

b. In 2002, there were 53.8 billion aluminum cans collected for recycling. Approximately how many pounds of aluminum were collected? *Explain* how you found your answer.

MIXED REVIEW

Evaluate the expression.

45. $|-3.2| - 2.8$ *(p. 80)*
46. $-6.1 - (-8.4)$ *(p. 80)*
47. $\sqrt{196}$ *(p. 110)*

Graph the equation.

48. $x = 0$ *(p. 215)*
49. $y = 8$ *(p. 215)*
50. $4x - 2y = 7$ *(p. 225)*
51. $-x + 5y = 1$ *(p. 225)*
52. $y = 2x - 7$ *(p. 244)*
53. $y = -\frac{3}{4}x + 2$ *(p. 244)*

PREVIEW
Prepare for Lesson 5.4 in Exs. 54–57.

Write an equation of the line that has the given characteristics.

54. Slope: -3; y-intercept: 5 *(p. 283)*
55. Slope: 8; passes through (2, 15) *(p. 292)*
56. Passes through $(0, -3)$, $(6, 1)$ *(p. 283)*
57. Passes through $(3, 3)$, $(6, -1)$ *(p. 292)*

Extension
Use after Lesson 5.3

Relate Arithmetic Sequences to Linear Functions

GOAL Identify, graph, and write the general form of arithmetic sequences.

Key Vocabulary
- sequence
- arithmetic sequence
- common difference

A **sequence** is an ordered list of numbers. The numbers in a sequence are called *terms*. In an **arithmetic sequence**, the difference between consecutive terms is constant. The constant difference is called the **common difference**.

An arithmetic sequence has the form $a_1, a_1 + d, a_1 + 2d, \ldots$ where a_1 is the first term and d is the common difference. For instance, if $a_1 = 2$ and $d = 6$, then the sequence $2, 2 + 6, 2 + 2(6), \ldots$ or $2, 8, 14, \ldots$ is arithmetic.

EXAMPLE 1 Identify an arithmetic sequence

Tell whether the sequence is arithmetic. If it is, find the next two terms.

a. $-4, 1, 6, 11, 16, \ldots$

b. $3, 5, 9, 15, 23, \ldots$

Solution

a. The first term is $a_1 = -4$. Find the differences of consecutive terms.

$a_2 - a_1 = 1 - (-4) = 5$ $\quad$ $a_3 - a_2 = 6 - 1 = 5$

$a_4 - a_3 = 11 - 6 = 5$ $\quad$ $a_5 - a_4 = 16 - 11 = 5$

▶ Because the terms have a common difference ($d = 5$), the sequence is arithmetic. The next two terms are $a_6 = 21$ and $a_7 = 26$.

b. The first term is $a_1 = 3$. Find the differences of consecutive terms.

$a_2 - a_1 = 5 - 3 = 2$ $\quad$ $a_3 - a_2 = 9 - 5 = 4$

$a_4 - a_3 = 15 - 9 = 6$ $\quad$ $a_5 - a_4 = 23 - 15 = 8$

▶ There is no common difference, so the sequence is not arithmetic.

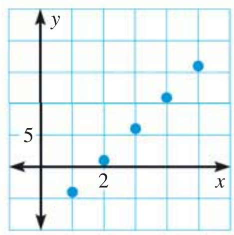
GRAPHING A SEQUENCE To graph a sequence, let a term's position number in the sequence be the x-value. The term is the corresponding y-value.

EXAMPLE 2 Graph a sequence

Graph the sequence $-4, 1, 6, 11, 16, \ldots$.

Make a table pairing each term with its position number.

Position, x	1	2	3	4	5
Term, y	-4	1	6	11	16



Plot the pairs in the table as points in a coordinate plane.

FUNCTIONS Notice that the points plotted in Example 2 appear to lie on a line. In fact, an arithmetic sequence is a linear function. You can think of the common difference d as the slope and $(1, a_1)$ as a point on the graph of the function. An equation in point-slope form for the function is $a_n - a_1 = d(n - 1)$. This equation can be rewritten as $a_n = a_1 + (n - 1)d$.

KEY CONCEPT *For Your Notebook*

Rule for an Arithmetic Sequence

The nth term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$.

EXAMPLE 3 Write a rule for the *n*th term of a sequence

Write a rule for the *n*th term of the sequence −4, 1, 6, 11, 16, ... Find a_{100}.

Solution

The first term of the sequence is $a_1 = -4$, and the common difference is $d = 5$.

$a_n = a_1 + (n - 1)d$ Write general rule for an arithmetic sequence.

$a_n = -4 + (n - 1)5$ Substitute −4 for a_1 and 5 for d.

Find a_{100} by substituting 100 for n.

$a_n = -4 + (n - 1)5$ Write the rule for the sequence.

$a_{100} = -4 + (100 - 1)5$ Substitute 100 for n.

$a_{100} = 491$ Evaluate.

PRACTICE

EXAMPLE 1 on p. 309 for Exs. 1–3

Tell whether the sequence is arithmetic. If it is, find the next two terms. If it is not, explain why not.

1. 17, 14, 11, 8, 5, . . .
2. 1, 4, 16, 64, 256, . . .
3. −8, −15, −22, −29, −36, . . .

EXAMPLE 2 on p. 309 for Exs. 4–9

Graph the sequence.

4. 1, 4, 7, 11, 14, . . .
5. 4, −3, −10, −17, −24, . . .
6. 5, −1, −7, −13, −19, . . .
7. $2, 3\frac{1}{2}, 5, 6\frac{1}{2}, 8, \ldots$
8. 0, 2, 4, 6, 8, . . .
9. −3, −4, −5, −6, −7, . . .

EXAMPLE 3 on p. 310 for Exs. 10–15

Write a rule for the *n*th term of the sequence. Find a_{100}.

10. −12, −5, 2, 9, 16, . . .
11. 51, 72, 93, 114, 135, . . .
12. 0.25, −0.75, −1.75, −2.75, . . .
13. $\frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \ldots$
14. 0, −5, −10, −15, −20, . . .
15. $1, 1\frac{1}{3}, 1\frac{2}{3}, 2, 2\frac{1}{3}, \ldots$
16. **REASONING** For an arithmetic sequence with a first term of a_1 and a common difference of d, show that $a_{n+1} - a_n = d$.

5.4 Write Linear Equations in Standard Form

Before You wrote equations in point-slope form.

Now You will write equations in standard form.

Why? So you can find possible combinations of objects, as in Ex. 41.

Key Vocabulary
- **standard form,** *p. 215*

Recall that the linear equation $Ax + By = C$ is in standard form, where A, B, and C are real numbers and A and B are not both zero. All linear equations can be written in standard form.

EXAMPLE 1 Write equivalent equations in standard form

Write two equations in standard form that are equivalent to $2x - 6y = 4$.

Solution

To write one equivalent equation, multiply each side by 2.

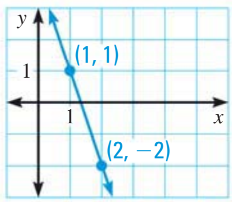
$$4x - 12y = 8$$

To write another equivalent equation, multiply each side by 0.5.

$$x - 3y = 2$$

EXAMPLE 2 Write an equation from a graph

Write an equation in standard form of the line shown.



Solution

STEP 1 **Calculate** the slope.

$$m = \frac{1 - (-2)}{1 - 2} = \frac{3}{-1} = -3$$

STEP 2 **Write** an equation in point-slope form. Use (1, 1).

$$y - y_1 = m(x - x_1) \qquad \text{Write point-slope form.}$$

$$y - 1 = -3(x - 1) \qquad \text{Substitute 1 for } y_1\text{, } -3 \text{ for } m\text{, and 1 for } x_1.$$

STEP 3 **Rewrite** the equation in standard form.

$$3x + y = 4 \qquad \text{Simplify. Collect variable terms on one side, constants on the other.}$$

Animated Algebra at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

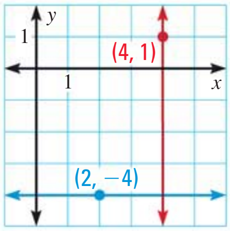
1. Write two equations in standard form that are equivalent to $x - y = 3$.
2. Write an equation in standard form of the line through $(3, -1)$ and $(2, -3)$.

HORIZONTAL AND VERTICAL LINES Recall that equations of horizontal lines have the form $y = a$. Equations of vertical lines have the form $x = b$. You cannot write an equation for a vertical line in slope-intercept form or point-slope form, because a vertical line has no slope. However, you can write an equation for a vertical line in standard form.

EXAMPLE 3 Write an equation of a line

Write an equation of the specified line.

a. Blue line **b.** Red line



Solution

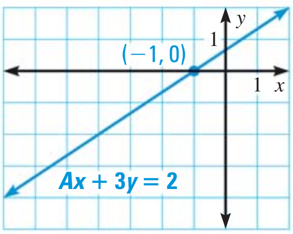
a. The y-coordinate of the given point on the blue line is -4. This means that all points on the line have a y-coordinate of -4. An equation of the line is $y = -4$.

b. The x-coordinate of the given point on the red line is 4. This means that all points on the line have an x-coordinate of 4. An equation of the line is $x = 4$.

ANOTHER WAY
Using the slope-intercept form to find an equation of the horizontal line gives you $y = 0x - 4$, or $y = -4$.

EXAMPLE 4 Complete an equation in standard form

Find the missing coefficient in the equation of the line shown. Write the completed equation.



Solution

STEP 1 **Find** the value of A. Substitute the coordinates of the given point for x and y in the equation. Solve for A.

$Ax + 3y = 2$ **Write equation.**

$A(-1) + 3(0) = 2$ **Substitute −1 for *x* and 0 for *y*.**

$-A = 2$ **Simplify.**

$A = -2$ **Divide by −1.**

STEP 2 **Complete** the equation.

$-2x + 3y = 2$ **Substitute −2 for *A*.**

✓ GUIDED PRACTICE for Examples 3 and 4

Write equations of the horizontal and vertical lines that pass through the given point.

3. $(-8, -9)$

4. $(13, -5)$

Find the missing coefficient in the equation of the line that passes through the given point. Write the completed equation.

5. $-4x + By = 7, (-1, 1)$

6. $Ax + y = -3, (2, 11)$

EXAMPLE 5 Solve a multi-step problem

LIBRARY Your class is taking a trip to the public library. You can travel in small and large vans. A small van holds 8 people and a large van holds 12 people. Your class could fill 15 small vans and 2 large vans.

a. **Write** an equation in standard form that models the possible combinations of small vans and large vans that your class could fill.

b. **Graph** the equation from part (a).

c. **List** several possible combinations.

Solution

a. Write a verbal model. Then write an equation.

Capacity of small van	$\cdot$	Number of small vans	$+$	Capacity of large van	$\cdot$	Number of large vans	$=$	People on trip
8	$\cdot$	s	$+$	12	$\cdot$	ℓ	$=$	p

Because your class could fill 15 small vans and 2 large vans, use (15, 2) as the s- and ℓ-values to substitute in the equation $8s + 12\ell = p$ to find the value of p.

$8(15) + 12(2) = p$ Substitute 15 for s and 2 for ℓ.

$144 = p$ Simplify.

Substitute 144 for p in the equation $8s + 12\ell = p$.

▶ The equation $8s + 12\ell = 144$ models the possible combinations.

b. Find the intercepts of the graph.

Substitute 0 for s.

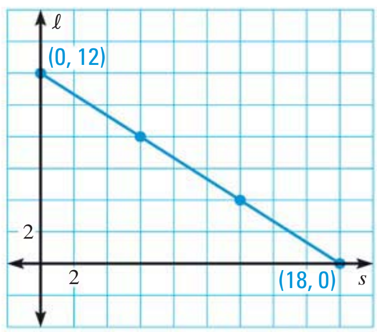
$8(0) + 12\ell = 144$

$\ell = 12$

Substitute 0 for ℓ.

$8s + 12(0) = 144$

$s = 18$



Plot the points (0, 12) and (18, 0). Connect them with a line segment. For this problem only nonnegative whole-number values of s and ℓ make sense.

c. The graph passes through (0, 12), (6, 8),(12, 4), and (18, 0). So, four possible combinations are 0 small and 12 large, 6 small and 8 large, 12 small and 4 large, 8 small and 0 large.

LISTING COMBINATIONS

Other combinations of small and large vans are possible. Another way to find possible combinations is by substituting values for s or ℓ in the equation.

✓ GUIDED PRACTICE for Example 5

7. **WHAT IF?** In Example 5, suppose that 8 students decide not to go on the class trip. Write an equation that models the possible combinations of small and large vans that your class could fill. List several possible combinations.

5.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 17 and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 4, 30, 40, and 42

◆ = **MULTIPLE REPRESENTATIONS** Ex. 41

SKILL PRACTICE

VOCABULARY **Identify the form of the equation.**

1. $2x + 8y = -3$
2. $y = -5x + 8$
3. $y + 4 = 2(x - 6)$

4. ★ **WRITING** *Explain* how to write an equation of a line in standard form when two points on the line are given.

EXAMPLE 1 on p. 311 for Exs. 5–10

EQUIVALENT EQUATIONS **Write two equations in standard form that are equivalent to the given equation.**

5. $x + y = -10$
6. $5x + 10y = 15$
7. $-x + 2y = 9$
8. $-9x - 12y = 6$
9. $9x - 3y = -12$
10. $-2x + 4y = -5$

EXAMPLE 2 on p. 311 for Exs. 11–22

WRITING EQUATIONS **Write an equation in standard form of the line that passes through the given point and has the given slope *m* or that passes through the two given points.**

11. $(-3, 2), m = 1$
12. $(4, -1), m = 3$
13. $(0, 5), m = -2$
14. $(-8, 0), m = -4$
15. $(-4, -4), m = -\frac{3}{2}$
16. $(-6, -10), m = \frac{1}{6}$
17. $(-8, 4), (4, -4)$
18. $(-5, 2), (-4, 3)$
19. $(0, -1), (-6, -9)$
20. $(3, 9), (1, 1)$
21. $(10, 6), (-12, -5)$
22. $(-6, -2), (-1, -2)$

EXAMPLE 3 on p. 312 for Exs. 23–28

HORIZONTAL AND VERTICAL LINES **Write equations of the horizontal and vertical lines that pass through the given point.**

23. $(3, 2)$
24. $(-5, -3)$
25. $(-1, 3)$
26. $(5, 3)$
27. $(-1, 4)$
28. $(-6, -2)$

EXAMPLE 4 on p. 312 for Exs. 29–36

29. **ERROR ANALYSIS** *Describe* and correct the error in finding the value of A for the equation $Ax - 3y = 5$, if the graph of the equation passes through the point $(1, -4)$.

$$A(-4) - 3(1) = 5$$
$$A = -2$$

30. ★ **MULTIPLE CHOICE** The graph of the equation $Ax + 2y = -2$ is a line that passes through $(2, -2)$. What is the value of A?

(A) -1 (B) 1 (C) 2 (D) 3

COMPLETING EQUATIONS **Find the missing coefficient in the equation of the line that passes through the given point. Write the completed equation.**

31. $Ax + 3y = 5, (2, -1)$
32. $Ax - 4y = -1, (6, 1)$
33. $-x + By = 10, (-2, -2)$
34. $8x + By = 4, (-5, 4)$
35. $Ax - 3y = -5, (1, 0)$
36. $2x + By = -4, (-3, 7)$

37. **CHALLENGE** Write an equation in standard form of the line that passes through $(0, a)$ and $(b, 0)$ where $a \neq 0$ and $b \neq 0$.

PROBLEM SOLVING

EXAMPLE 5
on p. 313
for Exs. 38–41

38. GARDENING The diagram shows the prices of two types of ground cover plants. Write an equation in standard form that models the possible combinations of vinca and phlox plants a gardener can buy for $300. List three of these possible combinations.

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39. NUTRITION A snack mix requires a total of 120 ounces of some corn cereal and some wheat cereal. Corn cereal comes in 12 ounce boxes.

a. The last time you made this mix, you used 5 boxes of corn cereal and 4 boxes of wheat cereal. How many ounces are in a box of wheat cereal?

b. Write an equation in standard form that models the possible combinations of boxes of wheat and corn cereal you can use.

c. List all possible combinations of whole boxes of wheat and corn cereal you can use to make the snack mix.

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40. ★ **SHORT RESPONSE** A dog kennel charges $20 per night to board your dog. You can also have a doggie treat delivered to your dog for $5. Write an equation that models the possible combinations of nights at the kennel and doggie treats that you can buy for $100. Graph the equation. *Explain* what the intercepts of the graph mean in this situation.

41. MULTIPLE REPRESENTATIONS As the student council treasurer, you prepare the budget for your class rafting trip. Each large raft costs $100 to rent, and each small raft costs $40 to rent. You have $1600 to spend.

a. Writing an Equation Write an equation in standard form that models the possible combinations of small rafts and large rafts that you can rent.

b. Drawing a Graph Graph the equation from part (a).

c. Making a Table Make a table that shows several combinations of small and large rafts that you can rent.

42. ★ **SHORT RESPONSE** One bus ride costs $.75. One subway ride costs $1.00. A monthly pass can be used for unlimited subway and bus rides and costs the same as 36 subway rides plus 36 bus rides.

a. Write an equation in standard form that models the possible combinations of bus and subway rides with the same value as the pass.

b. You ride the bus 60 times in one month. How many times must you ride the subway in order for the cost of the rides to equal the value of the pass? *Explain* your answer.

43. **GEOMETRY** Write an equation in standard form that models the possible lengths and widths (in feet) of a rectangle having the same perimeter as a rectangle that is 10 feet wide and 20 feet long. Make a table that shows five possible lengths and widths of the rectangle.

44. **CHALLENGE** You are working in a chemistry lab. You have 1000 milliliters of pure acid. A dilution of acid is created by adding pure acid to water. A 40% dilution contains 40% acid and 60% water. You have been asked to make a 40% dilution and a 60% dilution of pure acid.
   a. Write an equation in standard form that models the possible quantities of each dilution you can prepare using all 1000 milliliters of pure acid.
   b. You prepare 700 milliliters of the 40% dilution. How much of the 60% dilution can you prepare?
   c. How much water do you need to prepare 700 milliliters of the 40% dilution?

MIXED REVIEW

PREVIEW
Prepare for Lesson 5.5 in Exs. 45–46.

Tell whether the graphs of the two equations are parallel lines. *Explain* your reasoning. *(p. 244)*

45. $1 - y = 4x, -6 = -4x - y$

46. $4x = 2y - 6, 4 + y = -2x$

Write an equation in point-slope form of the line that passes through the given point and has the given slope *m*. *(p. 302)*

47. $(3, -4), m = 1$

48. $(-6, 6), m = -2$

49. $(-8, -1), m = 5$

QUIZ *for Lessons 5.1–5.4*

Write an equation in slope-intercept form of the line that passes through the given point and has the given slope *m*.

1. $(2, 5), m = 3$ *(p. 292)*

2. $(-1, 4), m = -2$ *(p. 292)*

3. $(0, -7), m = 5$ *(p. 283)*

Write an equation in slope-intercept form of the line that passes through the given points.

4. $(0, 2), (9, 5)$ *(p. 283)*

5. $(5, 7), (19, 14)$ *(p. 292)*

6. $(4, 24), (-11, 19)$ *(p. 292)*

Write an equation in (a) point-slope form and (b) standard form of the line that passes through the given points. *(pp. 302, 311)*

7. $(-5, 2), (-4, 3)$

8. $(0, -1), (-6, -9)$

9. $(3, 9), (1, 1)$

10. **DVDS** The table shows the price per DVD for different quantities of DVDs. Write an equation that models the price per DVD as a function of the number of DVDs purchased. *(p. 302)*

Number of DVDs purchased	1	2	3	4	5	6
Price per DVD (dollars)	20	18	16	14	12	10

EXTRA PRACTICE for Lesson 5.4, p. 942 **ONLINE QUIZ** at classzone.com

MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
classzone.com

Lessons 5.1–5.4

1. **MULTI-STEP PROBLEM** A satellite radio company charges a monthly fee of $13 for service. To use the service, you must first buy equipment that costs $100.
   a. Identify the rate of change and starting value in this situation.
   b. Write an equation that gives the total cost of satellite radio as a function of the number of months of service.
   c. Find the total cost after 1 year of satellite radio service.

2. **MULTI-STEP PROBLEM** You hike 5 miles before taking a break. After your break, you continue to hike at an average speed of 3.5 miles per hour.

   a. Write an equation that gives the distance (in miles) that you hike as a function of the time (in hours) since your break.
   b. You hike for 4 hours after your break. Find the total distance you hike for the day.

3. **EXTENDED RESPONSE** The table shows the cost of a catered lunch buffet for different numbers of people.

Number of people	Cost (dollars)
12	192
18	288
24	384
30	480
36	576
42	672

   a. *Explain* why the situation can be modeled by a linear equation.
   b. Write an equation that gives the cost of the lunch buffet as a function of the number of people attending.
   c. What is the cost of a lunch buffet for 120 people?

4. **SHORT RESPONSE** You use a garden hose to fill a swimming pool at a constant rate. The pool is empty when you begin to fill it. The pool contains 15 gallons of water after 5 minutes. After 30 minutes, the pool contains 90 gallons of water. Write an equation that gives the volume (in gallons) of water in the pool as a function of the number of minutes since you began filling it. *Explain* how you can find the time it takes to put 150 gallons of water in the pool.

5. **EXTENDED RESPONSE** A city is paving a bike path. The same length of path is paved each day. After 4 days, there are 8 miles of path remaining to be paved. After 6 more days, there are 5 miles of path remaining to be paved.
   a. *Explain* how you know the situation can be modeled by a linear equation.
   b. Write an equation that gives the distance (in miles) remaining to be paved as a function of the number of days since the project began.
   c. In how many more days will the entire path be paved?

6. **OPEN-ENDED** Write an equation in standard form that models the possible combinations of nickels and dimes worth a certain amount of money (in dollars). List several of these possible combinations.

7. **GRIDDED ANSWER** You are saving money to buy a stereo system. You have saved $50 so far. You plan to save $20 each week for the next few months. How much money do you expect to have saved in 7 weeks?

8. **GRIDDED ANSWER** The cost of renting a moving van for a 26 mile trip is $62.50. The cost of renting the same van for a 38 mile trip is $65.50. The cost changes at a constant rate with respect to the length (in miles) of the trip. Find the total cost of renting the van for a 54 mile trip.

Investigating Algebra ACTIVITY *Use before Lesson 5.5*

5.5 If–Then Statements and Their Converses

MATERIALS • index cards

QUESTION Is the converse of a conditional statement true?

In Lesson 2.1, you learned that an if-then statement is a form of a conditional statement where the *if* part contains the hypothesis and the *then* part contains the conclusion. The *converse* of an if-then statement interchanges the hypothesis and conclusion of the original statement.

EXPLORE Write the converse

STEP 1 ***Make cards***

Write each phrase below on a separate index card.

it swims	it is a tree	it flies	it needs water	it has wings
it is a duck	it grows	it is a bird	it is an airplane	it is a frog

STEP 2 ***Write the conditional statement***

Place the cards face down. Select a card at random to be the hypothesis. Select another card at random to be the conclusion. Write the statement and determine whether it is true or false. If it is false, give a counterexample.

Hypothesis: it is a duck Conclusion: it has wings

Statement: If it is a duck, then it has wings.

The statement is true. All ducks have wings.

STEP 3 ***Write the converse***

Switch the order of the cards to create the converse statement. Determine whether the converse is true or false. If it is false, give a counterexample.

Hypothesis: it has wings Conclusion: it is a duck

Statement: If it has wings, then it is a duck.

The statement is false. Airplanes have wings, but they are not ducks.

STEP 4 ***Repeat***

Repeat Steps 2 and 3 ten times. Keep a record of your conditional statements and their converses.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. **REASONING** If a conditional statement is true, can you be sure that its converse is true? *Justify* your answer.
2. **REASONING** If the converse of a statement is true, can you be sure that the original statement is true? *Justify* your answer.

5.5 Write Equations of Parallel and Perpendicular Lines

Before	You used slope to determine whether lines are parallel.
Now	You will write equations of parallel and perpendicular lines.
Why?	So you can analyze growth rates, as in Ex. 33.

Key Vocabulary
- **converse**
- **perpendicular lines**
- **conditional statement,** *p. 66*

The **converse** of a conditional statement interchanges the hypothesis and conclusion. The converse of a true statement is not necessarily true.

In Chapter 4, you learned that the statement "If two nonvertical lines have the same slope, then they are parallel" is true. Its converse is also true.

KEY CONCEPT *For Your Notebook*

Parallel Lines

- If two nonvertical lines in the same plane have the same slope, then they are parallel.
- If two nonvertical lines in the same plane are parallel, then they have the same slope.

EXAMPLE 1 Write an equation of a parallel line

Write an equation of the line that passes through (−3, −5) and is parallel to the line $y = 3x - 1$.

Solution

STEP 1 **Identify** the slope. The graph of the given equation has a slope of 3. So, the parallel line through (−3, −5) has a slope of 3.

STEP 2 **Find** the y-intercept. Use the slope and the given point.

$y = mx + b$	Write slope-intercept form.
$-5 = 3(-3) + b$	Substitute 3 for m, −3 for x, and −5 for y.
$4 = b$	Solve for b.

STEP 3 **Write** an equation. Use $y = mx + b$.

$y = 3x + 4$	Substitute 3 for m and 4 for b.

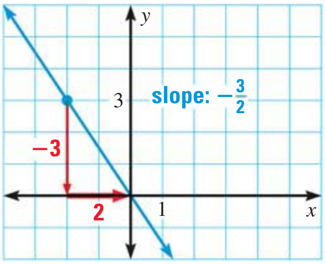
CHECK REASONABLENESS
You can check that your answer is reasonable by graphing both lines.

✓ GUIDED PRACTICE for Example 1

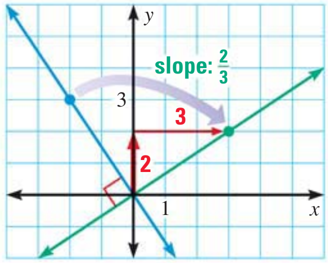
1. Write an equation of the line that passes through (−2, 11) and is parallel to the line $y = -x + 5$.

PERPENDICULAR LINES Two lines in the same plane are **perpendicular** if they intersect to form a right angle. Horizontal and vertical lines are perpendicular to each other.

Compare the slopes of the perpendicular lines shown below.



Rotate the line 90° in a clockwise direction about the origin to find a perpendicular line.



KEY CONCEPT *For Your Notebook*

Perpendicular Lines

- If two nonvertical lines in the same plane have slopes that are negative reciprocals, then the lines are perpendicular.
- If two nonvertical lines in the same plane are perpendicular, then their slopes are negative reciprocals.

USE FRACTIONS
The product of a nonzero number m and its negative reciprocal is −1: $m\left(-\frac{1}{m}\right) = -1$.

EXAMPLE 2 Determine whether lines are parallel or perpendicular

Determine which lines, if any, are parallel or perpendicular.

Line *a*: $y = 5x - 3$ **Line *b*:** $x + 5y = 2$ **Line *c*:** $-10y - 2x = 0$

Solution

Find the slopes of the lines.

Line *a*: The equation is in slope-intercept form. The slope is 5.

Write the equations for lines *b* and *c* in slope-intercept form.

Line *b*: $x + 5y = 2$

$$5y = -x + 2$$

$$y = -\frac{1}{5}x + \frac{2}{5}$$

Line *c*: $-10y - 2x = 0$

$$-10y = 2x$$

$$y = -\frac{1}{5}x$$

▶ Lines *b* and *c* have slopes of $-\frac{1}{5}$, so they are parallel. Line *a* has a slope of 5, the negative reciprocal of $-\frac{1}{5}$, so it is perpendicular to lines *b* and *c*.

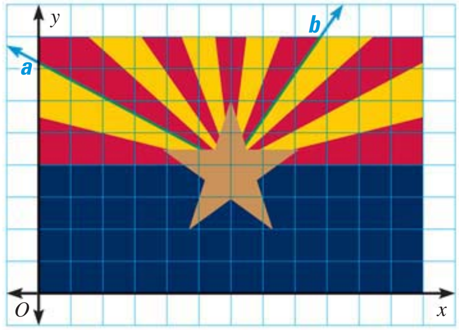
✓ **GUIDED PRACTICE** for Example 2

2. Determine which lines, if any, are parallel or perpendicular.

   Line *a*: $2x + 6y = -3$ **Line *b*:** $y = 3x - 8$ **Line *c*:** $-1.5y + 4.5x = 6$

EXAMPLE 3 Determine whether lines are perpendicular

STATE FLAG The Arizona state flag is shown in a coordinate plane. Lines a and b appear to be perpendicular. Are they?



Line *a*: $12y = -7x + 42$

Line *b*: $11y = 16x - 52$

Solution

Find the slopes of the lines. Write the equations in slope-intercept form.

Line *a*: $12y = -7x + 42$ **Line *b*:** $11y = 16x - 52$

$y = -\frac{7}{12}x + \frac{42}{12}$ $y = \frac{16}{11}x - \frac{52}{11}$

▶ The slope of line a is $-\frac{7}{12}$. The slope of line b is $\frac{16}{11}$. The two slopes are not negative reciprocals, so lines a and b are not perpendicular.

EXAMPLE 4 Write an equation of a perpendicular line

Write an equation of the line that passes through (4, −5) and is perpendicular to the line $y = 2x + 3$.

Solution

STEP 1 **Identify** the slope. The graph of the given equation has a slope of 2. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line through (4, −5) is $-\frac{1}{2}$.

STEP 2 **Find** the y-intercept. Use the slope and the given point.

$y = mx + b$ **Write slope-intercept form.**

$-5 = -\frac{1}{2}(4) + b$ **Substitute $-\frac{1}{2}$ for m, 4 for x, and −5 for y.**

$-3 = b$ **Solve for b.**

STEP 3 **Write** an equation.

$y = mx + b$ **Write slope-intercept form.**

$y = -\frac{1}{2}x - 3$ **Substitute $-\frac{1}{2}$ for m and −3 for b.**

✓ GUIDED PRACTICE for Examples 3 and 4

3. Is line a perpendicular to line b? *Justify* your answer using slopes.

   Line *a*: $2y + x = -12$ **Line *b*:** $2y = 3x - 8$

4. Write an equation of the line that passes through (4, 3) and is perpendicular to the line $y = 4x - 7$.

5.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 19 and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 17, 28, 30, 34, and 36

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Two lines in a plane are _?_ if they intersect to form a right angle.

2. ★ **WRITING** *Explain* how you can tell whether two lines are perpendicular, given the equations of the lines.

EXAMPLE 1 on p. 319 for Exs. 3–11

PARALLEL LINES Write an equation of the line that passes through the given point and is parallel to the given line.

3. $(-1, 3)$, $y = 2x + 2$
4. $(6, 8)$, $y = -\frac{5}{2}x + 10$
5. $(5, -1)$, $y = -\frac{3}{5}x - 3$
6. $(-1, 2)$, $y = 5x + 4$
7. $(1, 7)$, $-6x + y = -1$
8. $(18, 2)$, $3y = x - 12$
9. $(-2, 5)$, $2y = 4x - 6$
10. $(9, 4)$, $y - x = 3$
11. $(-10, 0)$, $-y + 3x = 16$

EXAMPLE 2 on p. 320 for Exs. 12–16

PARALLEL OR PERPENDICULAR Determine which lines, if any, are parallel or perpendicular.

12. Line a: $y = 4x - 2$, Line b: $y = -\frac{1}{4}x$, Line c: $y = -4x + 1$
13. Line a: $y = \frac{3}{5}x + 1$, Line b: $5y = 3x - 2$, Line c: $10x - 6y = -4$
14. Line a: $y = 3x + 6$, Line b: $3x + y = 6$, Line c: $3y = 2x + 18$
15. Line a: $4x - 3y = 2$, Line b: $3x + 4y = -1$, Line c: $4y - 3x = 20$

16. ★ **MULTIPLE CHOICE** Which statement is true of the given lines?

    Line a: $-2x + y = 4$ Line b: $2x + 5y = 2$ Line c: $x + 2y = 4$

    (A) Lines a and b are parallel.
    (B) Lines a and c are parallel.
    (C) Lines a and b are perpendicular.
    (D) Lines a and c are perpendicular.

17. ★ **SHORT RESPONSE** Determine which of the lines shown, if any, are parallel or perpendicular. *Justify* your answer using slopes.

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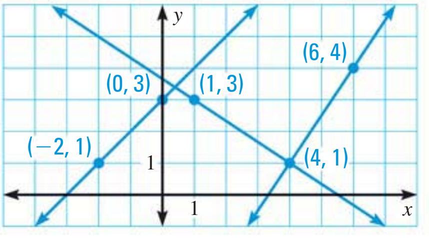


EXAMPLE 4 on p. 321 for Exs. 18–27

PERPENDICULAR LINES Write an equation of the line that passes through the given point and is perpendicular to the given line.

18. $(3, -3)$, $y = x + 5$
19. $(-9, 2)$, $y = 3x - 12$
20. $(5, 1)$, $y = 5x - 2$
21. $(7, 10)$, $y = 0.5x - 9$
22. $(-2, -4)$, $y = -\frac{2}{7}x + 1$
23. $(-4, -1)$, $y = \frac{4}{3}x + 6$
24. $(3, 3)$, $2y = 3x - 6$
25. $(-5, 2)$, $y + 3 = 2x$
26. $(8, -1)$, $4y + 2x = 12$

27. **ERROR ANALYSIS** *Describe* and correct the error in finding the y-intercept of the line that passes through $(2, 1)$ and is perpendicular to the line $y = -\frac{1}{2}x + 3$.

$y = mx + b$
$2 = 2(1) + b$
$0 = b$

28. ★ **MULTIPLE CHOICE** Which equation represents the line that passes through $(0, 0)$ and is parallel to the line passing through $(2, 3)$ and $(6, 1)$?

Ⓐ $y = \frac{1}{2}x$ Ⓑ $y = -\frac{1}{2}x$ Ⓒ $y = -2x$ Ⓓ $y = 2x$

29. **REASONING** Is the line through $(4, 3)$ and $(3, -1)$ perpendicular to the line through $(-3, 3)$ and $(1, 2)$? *Justify* your answer using slopes.

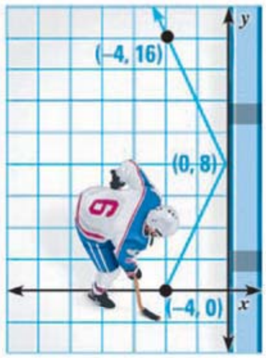
30. ★ **OPEN-ENDED** Write equations of two lines that are parallel. Then write an equation of a line that is perpendicular to those lines.

31. **CHALLENGE** Write a formula for the slope of a line that is perpendicular to the line through the points (x_1, y_1) and (x_2, y_2).

PROBLEM SOLVING

EXAMPLES 3 and 4 on p. 321 for Exs. 32, 34

32. **HOCKEY** A hockey puck leaves the blade of a hockey stick, bounces off a wall, and travels in a new direction, as shown.



a. Write an equation that models the path of the puck from the blade of the hockey stick to the wall.

b. Write an equation that models the path of the puck after it bounces off the wall.

c. Does the path of the puck form a right angle? *Justify* your answer.

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33. **BIOLOGY** While nursing, blue whale calves can gain weight at a rate of 200 pounds per day. Two particular calves weigh 6000 pounds and 6250 pounds at birth.

a. Write equations that model the weight of each calf as a function of the number of days since birth.

b. How much is each calf expected to weigh 30 days after birth?

c. How are the graphs of the equations from part (a) related? *Justify* your answer.

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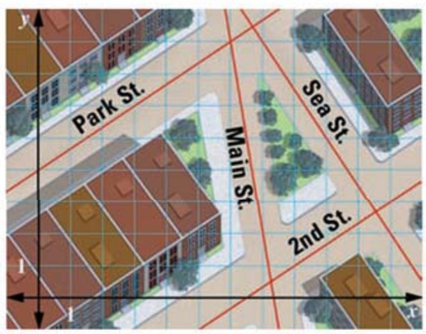
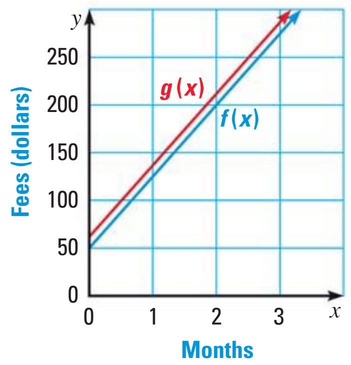
34. ★ **SHORT RESPONSE** The map shows several streets in a city. Determine which of the streets, if any, are parallel or perpendicular. *Jusify* your answer using slopes.



Park: $3y - 2x = 12$ Main: $y = -6x + 44$

2nd St.: $3y = 2x - 13$ Sea: $2y = -3x + 37$

35. **SOFTBALL** A softball training academy charges students a monthly fee plus an initial registration fee. The total amounts paid by two students are given by the functions $f(x)$ and $g(x)$ where x is the numbers of months the students have been members of the academy. The graphs of f and g are parallel lines. Did the students pay different monthly fees or different registration fees? How do you know?



36. ★ **EXTENDED RESPONSE** If you are one of the first 100 people to join a new health club, you are charged a joining fee of \$49. Otherwise, you are charged a joining fee of \$149. The monthly membership cost is \$38.75.

   a. Write an equation that gives the total cost (in dollars) of membership as a function of the number of months of membership if you are one of the first 100 members to join.

   b. Write an equation that gives the total cost (in dollars) of membership as a function of the number of months of membership if you are *not* one of the first 100 members to join.

   c. How are the graphs of these functions related? How do you know?

   d. After 6 months, what is the difference in total cost for a person who paid \$149 to join and a person who paid \$49 to join? after 12 months?

37. **CHALLENGE** You and your friend have gift cards to a shopping mall. Your card has a value of \$50, and your friend's card has a value of \$30. If neither of you uses the cards, the value begins to decrease at a rate of \$2.50 per month after 6 months.

   a. Write two equations, one that gives the value of your card and another that gives the value of your friend's card as functions of the number of months after 6 months of nonuse.

   b. How are the graphs of these functions related? How do you know?

   c. What are the x-intercepts of the graphs of the functions, and what do they mean in this situation?

MIXED REVIEW

Solve the equation or proportion.

38. $5z + 6z = 77$ *(p. 141)* 39. $-8n = 4(3n + 5)$ *(p. 148)* 40. $\frac{3}{5} = \frac{t}{7}$ *(p. 162)*

PREVIEW
Prepare for Lesson 5.6 in Ex. 41.

41. **CAMPING** The table shows the cost C (in dollars) for one person to stay at a campground for n nights. *(p. 253)*

Number of nights, n	1	3	5	9
Cost, C (in dollars)	15	45	75	135

   a. *Explain* why C varies directly with n.

   b. Write a direct variation equation that relates C and n.

42. Write an equation in standard form of the line that passes through the points $(3, -9)$ and $(12, 9)$. *(p. 311)*

5.6 Fit a Line to Data

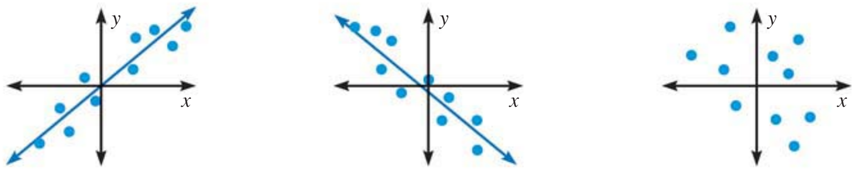
Before You modeled situations involving a constant rate of change.

Now You will make scatter plots and write equations to model data.

Why? So you can model scientific data, as in Ex. 19.

Key Vocabulary
- scatter plot
- correlation
- line of fit

A **scatter plot** is a graph used to determine whether there is a relationship between paired data. Scatter plots can show trends in the data.



If y tends to increase as x increases, the paired data are said to have a **positive correlation**.

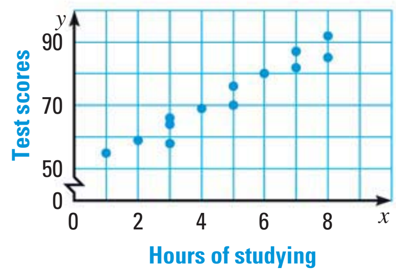
If y tends to decrease as x increases, the paired data are said to have a **negative correlation**.

If x and y have no apparent relationship, the paired data are said to have **relatively no correlation**.

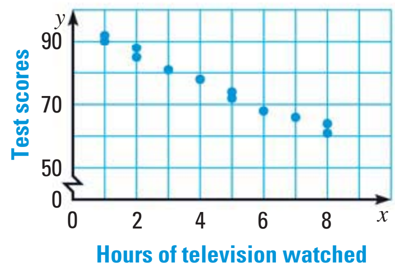
EXAMPLE 1 Describe the correlation of data

Describe **the correlation of the data graphed in the scatter plot.**

a.



b.



a. The scatter plot shows a positive correlation between hours of studying and test scores. This means that as the hours of studying increased, the test scores tended to increase.

b. The scatter plot shows a negative correlation between hours of television watched and test scores. This means that as the hours of television watched increased, the test scores tended to decrease.

✓ **GUIDED PRACTICE** for Example 1

1. Using the scatter plots in Example 1, predict a reasonable test score for 4.5 hours of studying and 4.5 hours of television watched.

EXAMPLE 2 Make a scatter plot

SWIMMING SPEEDS The table shows the lengths (in centimeters) and swimming speeds (in centimeters per second) of six fish.

Fish	Pike	Red gurnard	Black bass	Gurnard	Norway haddock
Length (cm)	37.8	19.2	21.3	26.2	26.8
Speed (cm/sec)	148	47	88	131	98

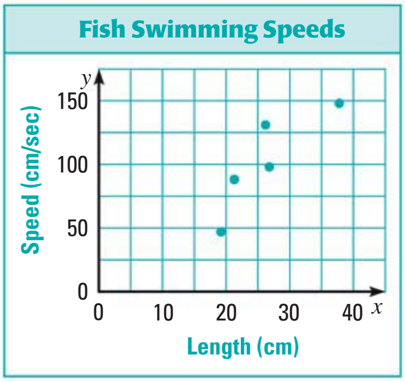
a. Make a scatter plot of the data.

b. *Describe* the correlation of the data.

Solution

a. Treat the data as ordered pairs. Let x represent the fish length (in centimeters), and let y represent the speed (in centimeters per second). Plot the ordered pairs as points in a coordinate plane.

b. The scatter plot shows a positive correlation, which means that longer fish tend to swim faster.



GUIDED PRACTICE for Example 2

2. Make a scatter plot of the data in the table. *Describe* the correlation of the data.

x	1	1	2	3	3	4	5	5	6
y	2	3	4	4	5	5	5	7	8

MODELING DATA When data show a positive or negative correlation, you can model the trend in the data using a **line of fit**.

KEY CONCEPT *For Your Notebook*

Using a Line of Fit to Model Data

STEP 1 **Make** a scatter plot of the data.

STEP 2 **Decide** whether the data can be modeled by a line.

STEP 3 **Draw** a line that appears to fit the data closely. There should be approximately as many points above the line as below it.

STEP 4 **Write** an equation using two points on the line. The points do not have to represent actual data pairs, but they must lie on the line of fit.

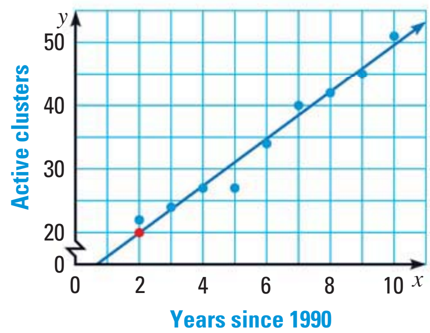
EXAMPLE 3 Write an equation to model data

BIRD POPULATIONS The table shows the number of active red-cockaded woodpecker clusters in a part of the De Soto National Forest in Mississippi. Write an equation that models the number of active clusters as a function of the number of years since 1990.

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Active clusters	22	24	27	27	34	40	42	45	51

Solution

STEP 1 **Make** a scatter plot of the data. Let x represent the number of years since 1990. Let y represent the number of active clusters.



STEP 2 **Decide** whether the data can be modeled by a line. Because the scatter plot shows a positive correlation, you can fit a line to the data.

STEP 3 **Draw** a line that appears to fit the points in the scatter plot closely.

STEP 4 **Write** an equation using two points on the line. Use (2, 20) and (8, 42).

Find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{42 - 20}{8 - 2} = \frac{22}{6} = \frac{11}{3}$$

Find the y-intercept of the line. Use the point (2, 20).

$y = mx + b$ **Write slope-intercept form.**

$20 = \frac{11}{3}(2) + b$ **Substitute $\frac{11}{3}$ for m, 2 for x, and 20 for y.**

$\frac{38}{3} = b$ **Solve for b.**

An equation of the line of fit is $y = \frac{11}{3}x + \frac{38}{3}$.

▶ The number y of active woodpecker clusters can be modeled by the function $y = \frac{11}{3}x + \frac{38}{3}$ where x is the number of years since 1990.

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✓ GUIDED PRACTICE for Example 3

3. Use the data in the table to write an equation that models y as a function of x.

x	1	2	3	4	5	6	8
y	3	5	8	9	11	12	14

EXAMPLE 4 Interpret a model

Refer to the model for the number of woodpecker clusters in Example 3.

a. *Describe* the domain and range of the function.

b. At about what rate did the number of active woodpecker clusters change during the period 1992–2000?

Solution

a. The domain of the function is the the period from 1992 to 2000, or $2 \le x \le 10$. The range is the the number of active clusters given by the function for $2 \le x \le 10$, or $20 \le y \le 49.3$.

b. The number of active woodpecker clusters increased at a rate of $\frac{11}{3}$ or about 3.7 woodpecker clusters per year.

✓ GUIDED PRACTICE for Example 4

4. In Guided Practice Exercise 2, at about what rate does y change with respect to x?

5.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7 and 17

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 8, 11, 12, and 16

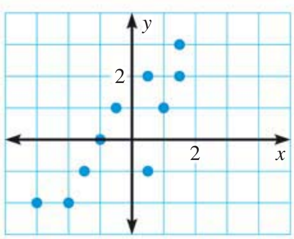
SKILL PRACTICE

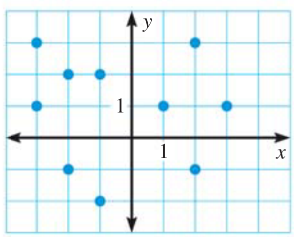
1. **VOCABULARY** Copy and complete: When data have a positive correlation, the dependent variable tends to _?_ as the independent variable increases.

2. ★ **WRITING** *Describe* how paired data with a positive correlation, a negative correlation, and relatively no correlation differ.

EXAMPLE 1 on p. 325 for Exs. 3–5, 10, 11

DESCRIBING CORRELATIONS Tell whether x and y show a *positive correlation*, a *negative correlation*, or *relatively no correlation*.

3.


4.


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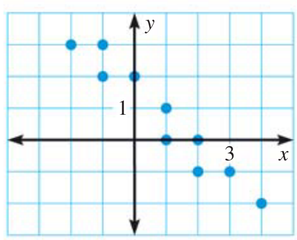


EXAMPLES 2 and 3 on pp. 326–327 for Exs. 6–9

FITTING LINES TO DATA Make a scatter plot of the data in the table. Draw a line of fit. Write an equation of the line.

6.

x	1	1	3	4	5	6	9
y	10	12	33	46	59	70	102

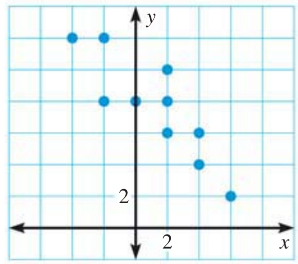
7.

x	1.2	1.8	2.3	3.0	4.4	5.2
y	10	7	5	−1	−4	−8

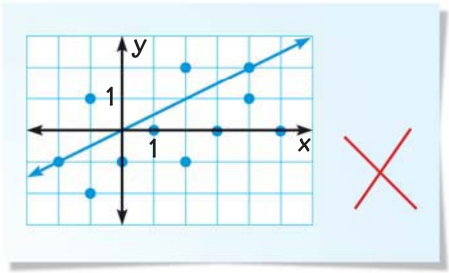
8. ★ **MULTIPLE CHOICE** Which equation best models the data in the scatter plot?

Ⓐ $y = -x - 6$ Ⓑ $y = x - 6$

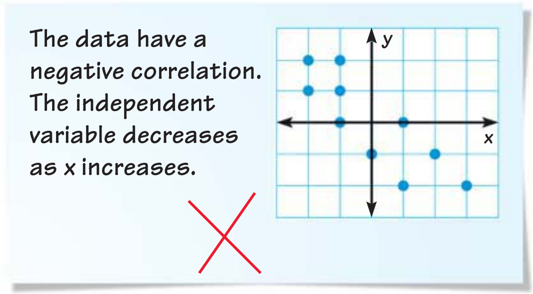
Ⓒ $y = -x + 8$ Ⓓ $y = x + 8$



9. **ERROR ANALYSIS** *Describe* and correct the error in fitting the line to the data in the scatter plot.



10. **ERROR ANALYSIS** *Describe* and correct the error in describing the correlation of the data in the scatter plot.



11. ★ **OPEN-ENDED** Give an example of a data set that shows a negative correlation.

12. ★ **SHORT RESPONSE** Make a scatter plot of the data. *Describe* the correlation of the data. Is it possible to fit a line to the data? If so, write an equation of the line. If not, explain why.

x	−12	−7	−4	−3	−1	2	5	6	7	9	15
y	150	50	15	10	1	5	22	37	52	90	226

MODELING DATA **Make a scatter plot of the data. *Describe* the correlation of the data. If possible, fit a line to the data and write an equation of the line.**

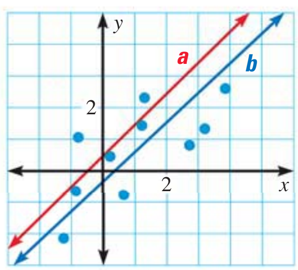
13.

x	10	12	15	20	30	45	60	99
y	−2	4	9	16	32	55	87	128

14.

x	−5	−3	−3	0	1	2	5	6
y	−4	12	10	−6	8	0	3	−9

15. **CHALLENGE** Which line shown is a better line of fit for the scatter plot? *Explain* your reasoning.



PROBLEM SOLVING

EXAMPLE 2
on p. 326
for Exs. 16

16. ★ **SHORT RESPONSE** The table shows the approximate home range size of big cats (members of the *Panthera* genus) in their natural habitat and the percent of time that the cats spend pacing in captivity.

Big cat (*Panthera* genus)	Lion	Jaguar	Leopard	Tiger
Home range size (km^2)	148	90	34	48
Pacing (percent of time)	48	21	11	16

a. Make a scatter plot of the data.

b. *Describe* the correlation of the data.

c. The snow leopard's home range size is about 39 square kilometers. It paces about 7% of its time in captivity. Does the snow leopard fit the pacing trend of cats in the *Panthera* genus? *Explain* your reasoning.

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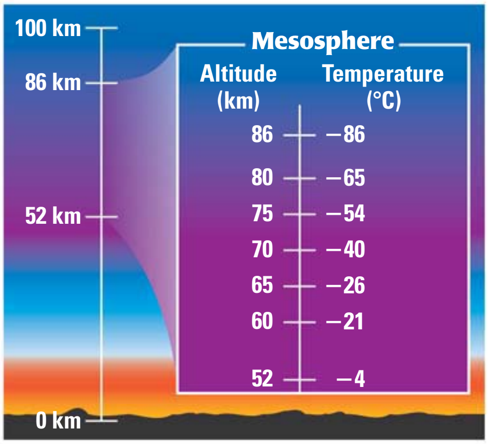
EXAMPLES 3 and 4
on pp. 327–328
for Exs. 17–18

17. **EARTH SCIENCE** The mesosphere is a layer of atmosphere that lies from about 50 kilometers above Earth's surface to about 90 kilometers above Earth's surface. The diagram shows the temperature at certain altitudes in the mesosphere.



a. Make a scatter plot of the data.

b. Write an equation that models the temperature (in degrees Celsius) as a function of the altitude (in kilometers) above 50 kilometers.

c. At about what rate does the temperature change with increasing altitude in the mesosphere?

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18. **ALLIGATORS** The table shows the weights of two alligators at various times during a feeding trial. Make two scatter plots, one for each alligator, where x is the number of weeks and y is the weight of the alligator. Draw lines of fit for both scatter plots. *Compare* the approximate growth rates.

Weeks	0	9	18	27	34	43	49
Alligator 1 weight (pounds)	6	8.6	10	13.6	15	17.2	19.8
Alligator 2 weight (pounds)	6	9.2	12.8	13.6	20.2	21.4	24.3

19. **GEOLOGY** The table shows the duration of several eruptions of the geyser Old Faithful and the interval between eruptions. Write an equation that models the interval as a function of an eruption's duration.

Duration (minutes)	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Interval (minutes)	50	57	65	71	76	82	89	95

◯ = WORKED-OUT SOLUTIONS on p. WS1　　★ = STANDARDIZED TEST PRACTICE

20. DAYLIGHT The table shows the number of hours and minutes of daylight in Baltimore, Maryland, for ten days in January.

Day in January	5	6	7	8	9	10	11	12	13	14
Daylight (hours and minutes)	9:30	9:31	9:32	9:34	9:35	9:36	9:37	9:38	9:40	9:41

a. Write an equation that models the hours of daylight (in minutes in excess of 9 hours) as a function of the number of days since January 5.

b. At what rate do the hours of daylight change over time in early January?

c. Do you expect the trend described by the equation to continue indefinitely? *Explain*.

21. CHALLENGE The table shows the estimated amount of time and the estimated amount of money the average person in the U.S. spent on the Internet each year from 1999 to 2005.

Year	1999	2000	2001	2002	2003	2004	2005
Internet time (hours)	88	107	136	154	169	182	193
Internet spending (dollars)	40.55	49.64	68.70	84.73	97.76	110.46	122.67

a. Write an equation that models the amount of time h (in hours) spent on the Internet as a function of the number of years y since 1999.

b. Write an equation that models the amount of money m spent on the Internet as a function of the time h (in hours) spent on the Internet.

c. Substitute the expression that is equal to h from part (a) in the function from part (b). What does the new function tell you?

d. Does the function from part (c) agree with the data given? *Explain*.

MIXED REVIEW

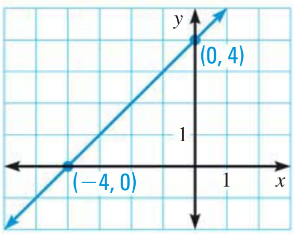
PREVIEW
Prepare for Lesson 5.7 in Exs. 22–24.

Evaluate the function when $x = -2, 5$, and 0. *(p. 262)*

22. $f(x) = 5x - 8$ **23.** $g(x) = -10x$ **24.** $v(x) = 14 - 5x$

Write an equation of the line shown. *(p. 283)*

25.



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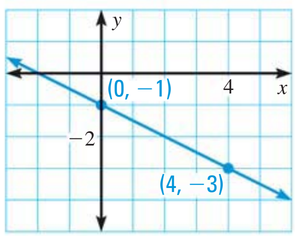
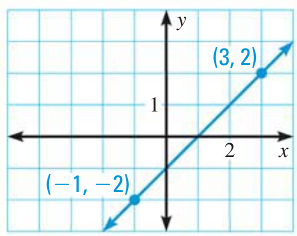


27.



28. Determine which lines, if any, are parallel or perpendicular. *(p. 319)*

Line a: $y = 2x - 5$ Line b: $2x + y = -5$ Line c: $4x - 2y = 3$

5.6 Perform Linear Regression

QUESTION How can you model data with the best-fitting line?

The line that most closely follows a trend in data is the *best-fitting line*. The process of finding the best-fitting line to model a set of data is called *linear regression*. This process can be tedious to perform by hand, but you can use a graphing calculator to make a scatter plot and perform linear regression on a data set.

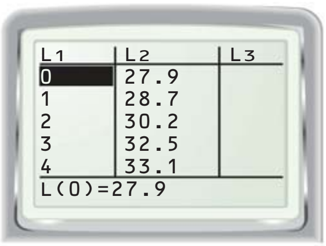
EXAMPLE 1 Create a scatter plot

The table shows the total sales from women's clothing stores in the United States from 1997 to 2002. Make a scatter plot of the data. *Describe* the correlation of the data.

Year	1997	1998	1999	2000	2001	2002
Sales (billions of dollars)	27.9	28.7	30.2	32.5	33.1	34.3

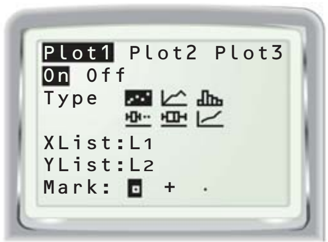
STEP 1 Enter data

Press STAT and select Edit. Enter years since 1997 (0, 1, 2, 3, 4, 5) into List 1 (L_1). These will be the x-values. Enter sales (in billions of dollars) into List 2 (L_2). These will be the y-values.



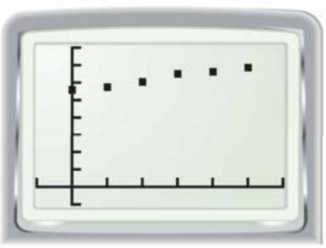
STEP 2 Choose plot settings

Press 2nd Y= and select Plot1. Turn Plot1 On. Select scatter plot as the type of display. Enter L_1 for the Xlist and L_2 for the Ylist.



STEP 3 Make a scatter plot

Press ZOOM 9 to display the scatter plot so that the points for all data pairs are visible.



STEP 4 Describe the correlation

Describe the correlation of the data in the scatter plot.

The data have a positive correlation. This means that with each passing year, the sales of women's clothing tended to increase.

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Keystrokes

MODELING DATA The *correlation coefficient r* for a set of paired data measures how well the best-fitting line fits the data. You can use a graphing calculator to find a value for r.

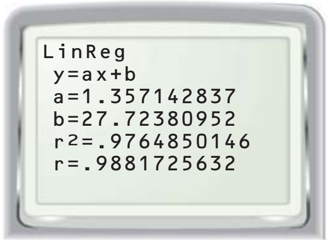
For r close to 1, the data have a strong positive correlation. For r close to -1, the data have a strong negative correlation. For r close to 0, the data have relatively no correlation.

EXAMPLE 2 Find the best-fitting line

Find an equation of the best-fitting line for the scatter plot from Example 1. Determine the correlation coefficient of the data. Graph the best-fitting line.

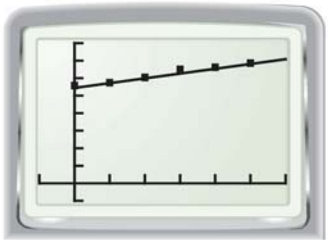
STEP 1 ***Perform regression***

Press STAT. From the CALC menu, choose LinReg(ax+b). The a- and b-values given are for an equation of the form $y = ax + b$. Rounding these values gives the equation $y = 1.36x + 27.7$. Because r is close to 1, the data have a strong positive correlation.



STEP 2 ***Draw the best-fitting line***

Press Y= and enter $1.36x + 27.7$ for y_1.
Press GRAPH.



PRACTICE

In Exercises 1–5, refer to the table, which shows the total sales from men's clothing stores in the United States from 1997 to 2002.

Year	1997	1998	1999	2000	2001	2002
Sales (billions of dollars)	10.1	10.6	10.5	10.8	10.3	9.9

1. Make a scatter plot of the data. *Describe* the correlation.
2. Find the equation of the best-fitting line for the data.
3. Draw the best-fitting line for the data.

DRAW CONCLUSIONS

4. What does the value of r for the equation in Exercise 2 tell you about the correlation of the data?
5. **PREDICT** How could you use the best-fitting line to predict future sales of men's clothing? *Explain* your answer.

5.7 Collecting and Organizing Data

MATERIALS • metric ruler

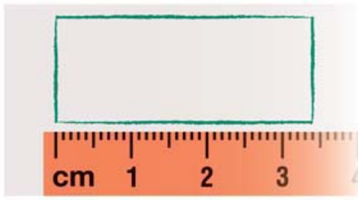
QUESTION How can you make a prediction using a line of fit?

EXPLORE Make a prediction using a line of fit

A student in your class draws a rectangle with a short side that is 4 centimeters in length. Predict the length of the long side of the rectangle.

STEP 1 *Collect data*

Ask each of 10 people to draw a rectangle. Do not let anyone drawing a rectangle see a rectangle drawn by someone else.



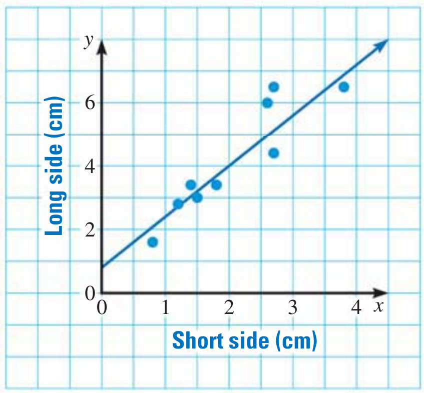
STEP 2 *Organize data*

Measure the lengths (in centimeters) of the short and long sides of the rectangles you collected. Create a table like the one shown.

Short side (cm)	2.7	2.7	1.8	2.6	1.4	1.5	1.2	0.8	3.8
Long side (cm)	4.4	6.5	3.4	6	3.4	3	2.8	1.6	6.5

STEP 3 *Graph data*

Make a scatter plot of the data where each point represents a rectangle that you collected. Let x represent the length of the short side of the rectangle, and let y represent the length of the long side.



STEP 4 *Model data*

Draw a line of fit.

STEP 5 *Predict*

Use the line of fit to find the length of the long side that corresponds to a short side with a length of 4 centimeters. In this case, the long side length predicted by the line of fit has a length of about 7 centimeters.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. **COMPARE** What is the slope of your line of fit? How does this slope compare with the slope of the line shown above?
2. **PREDICT** Suppose a student in your class draws a rectangle that has a long side with a length of 5 centimeters. Predict the length of the shorter side. *Explain* how you made your prediction.
3. **EXTEND** The *golden ratio* appears frequently in architectural structures, paintings, sculptures, and even in nature. This ratio of the long side of a rectangle to its short side is approximately 1.618. How does this ratio compare with the slopes of the lines you compared in Exercise 1?

5.7 Predict with Linear Models

Before You made scatter plots and wrote equations of lines of fit.

Now You will make predictions using best-fitting lines.

Why? So you can model trends, as in Ex. 21.

Key Vocabulary
- best-fitting line
- linear regression
- interpolation
- extrapolation
- zero of a function

The line that most closely follows a trend in data is called the **best-fitting line**. The process of finding the best-fitting line to model a set of data is called **linear regression**. You can perform linear regression using technology. Using a line or its equation to approximate a value between two known values is called **linear interpolation**.

EXAMPLE 1 Interpolate using an equation

CD SINGLES The table shows the total number of CD singles shipped (in millions) by manufacturers for several years during the period 1993–1997.

Year	1993	1995	1996	1997
CD singles shipped (millions)	7.8	22	43	67

REVIEW REGRESSION
For help with performing a linear regression to find the best-fitting line, see p. 332.

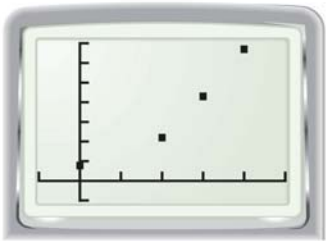
a. Make a scatter plot of the data.

b. Find an equation that models the number of CD singles shipped (in millions) as a function of the number of years since 1993.

c. Approximate the number of CD singles shipped in 1994.

Solution

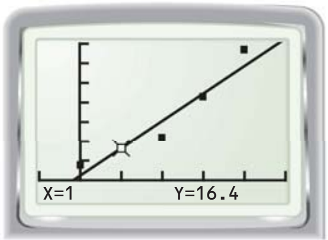
a. Enter the data into lists on a graphing calculator. Make a scatter plot, letting the number of years since 1993 be the x-values (0, 2, 3, 4) and the number of CD singles shipped be the y-values.



b. Perform linear regression using the paired data. The equation of the best-fitting line is approximately $y = 14x + 2.4$.

ANOTHER WAY
You can also estimate the number of CDs shipped in 1994 by evaluating $y = 14x + 2.4$ when $x = 1$.

c. Graph the best-fitting line. Use the *trace* feature and the arrow keys to find the value of the equation when $x = 1$.



▶ About 16 million CD singles were shipped in 1994.

Animated Algebra at classzone.com

EXTRAPOLATION Using a line or its equation to approximate a value outside the range of known values is called **linear extrapolation**.

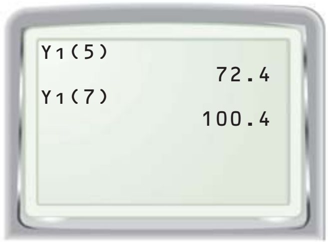
EXAMPLE 2 Extrapolate using an equation

CD SINGLES Look back at Example 1.

a. Use the equation from Example 1 to approximate the number of CD singles shipped in 1998 and in 2000.

b. In 1998 there were actually 56 million CD singles shipped. In 2000 there were actually 34 million CD singles shipped. *Describe* the accuracy of the extrapolations made in part (a).

Solution



a. Evaluate the equation of the best-fitting line from Example 1 for $x = 5$ and $x = 7$.

The model predicts about 72 million CD singles shipped in 1998 and about 100 million CD singles shipped in 2000.

b. The differences between the predicted number of CD singles shipped and the actual number of CD singles shipped in 1998 and 2000 are 16 million CDs and 66 million CDs, respectively. The difference in the actual and predicted numbers increased from 1998 to 2000. So, the equation of the best-fitting line gives a less accurate prediction for the year that is farther from the given years.

ACCURACY As Example 2 illustrates, the farther removed an x-value is from the known x-values, the less confidence you can have in the accuracy of the predicted y-value. This is true in general but not in every case.

✓ GUIDED PRACTICE for Examples 1 and 2

1. **HOUSE SIZE** The table shows the median floor area of new single-family houses in the United States during the period 1995–1999.

Year	1995	1996	1997	1998	1999
Median floor area (square feet)	1920	1950	1975	2000	2028

a. Find an equation that models the floor area (in square feet) of a new single-family house as a function of the number of years since 1995.

b. Predict the median floor area of a new single-family house in 2000 and in 2001.

c. Which of the predictions from part (b) would you expect to be more accurate? *Explain* your reasoning.

EXAMPLE 3 Predict using an equation

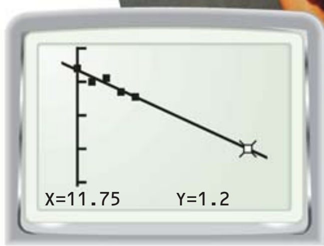
SOFTBALL The table shows the number of participants in U.S. youth softball during the period 1997–2001. Predict the year in which the number of youth softball participants reaches 1.2 million.

Year	1997	1998	1999	2000	2001
Participants (millions)	1.44	1.4	1.411	1.37	1.355

Solution

STEP 1 Perform linear regression. Let x represent the number of years since 1997, and let y represent the number of youth softball participants (in millions). The equation for the best-fitting line is approximately $y = -0.02x + 1.435$.

STEP 2 Graph the equation of the best-fitting line. Trace the line until the cursor reaches $y = 1.2$. The corresponding x-value is shown at the bottom of the calculator screen.



▶ There will be 1.2 million participants about 12 years after 1997, or in 2009.

ANOTHER WAY

You can also predict the year by substituting 1.2 for y in the equation and solving for x:

$$y = 0.02x + 1.435$$
$$1.2 = -0.02x + 1.435$$
$$x = 11.75$$

✓ GUIDED PRACTICE for Example 3

2. **SOFTBALL** In Example 3, in what year will there be 1.25 million youth softball participants in the U.S?

ZERO OF A FUNCTION A **zero of a function** $y = f(x)$ is an x-value for which $f(x) = 0$ (or $y = 0$). Because $y = 0$ along the x-axis of the coordinate plane, a zero of a function is an x-intercept of the function's graph.

KEY CONCEPT *For Your Notebook*

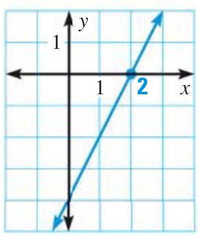
Relating Solutions of Equations, *x*-Intercepts of Graphs, and Zeros of Functions

In Chapter 3 you learned to solve an equation like $2x - 4 = 0$:

$$2x - 4 = 0$$
$$2x = 4$$
$$x = 2$$

The solution of $2x - 4 = 0$ is 2.

In Chapter 4 you found the x-intercept of the graph of a function like $y = 2x - 4$:



Now you are finding the zero of a function like $f(x) = 2x - 4$:

$$f(x) = 0$$
$$2x - 4 = 0$$
$$x = 2$$

The zero of $f(x) = 2x - 4$ is 2.

EXAMPLE 4 Find the zero of a function

SOFTBALL Look back at Example 3. Find the zero of the function. *Explain* what the zero means in this situation.

Solution

Substitute 0 for y in the equation of the best-fitting line and solve for x.

$y = -0.02x + 1.435$ **Write the equation.**

$0 = -0.02x + 1.435$ **Substitute 0 for y.**

$x \approx 72$ **Solve for x.**

▶ The zero of the function is about 72. The function has a negative slope, which means that the number of youth softball participants is decreasing. According to the model, there will be no youth softball participants 72 years after 1997, or in 2069.

✓ GUIDED PRACTICE for Example 4

3. **JET BOATS** The number y (in thousands) of jet boats purchased in the U.S. can be modeled by the function $y = -1.23x + 14$ where x is the number of years since 1995. Find the zero of the function. *Explain* what the zero means in this situation.

5.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3 and 19

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 16, and 21

◆ = **MULTIPLE REPRESENTATIONS** Exs. 22

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Using a linear function to approximate a value within a range of known data values is called __?__.

2. ★ **WRITING** *Explain* how extrapolation differs from interpolation.

EXAMPLE 1 on p. 335 for Exs. 3–4

LINEAR INTERPOLATION **Make a scatter plot of the data. Find the equation of the best-fitting line. Approximate the value of y for $x = 5$.**

3.

x	0	2	4	6	7
y	2	7	14	17	20

4.

x	2	4	6	8	10
y	6.2	22.5	40.2	55.4	72.1

EXAMPLE 2 on p. 336 for Exs. 5–6

LINEAR EXTRAPOLATION **Make a scatter plot of the data. Find the equation of the best-fitting line. Approximate the value of y for $x = 10$.**

5.

x	0	1	2	3	4
y	20	32	39	53	63

6.

x	1	3	5	7	9
y	0.4	1.4	1.9	2.3	3.2

EXAMPLE 4
on p. 338
for Exs. 7–13

ZERO OF A FUNCTION **Find the zero of the function.**

7. $f(x) = 7.5x - 20$ **8.** $f(x) = -x + 7$ **9.** $f(x) = \frac{1}{8}x + 2$

10. $f(x) = 17x - 68$ **11.** $f(x) = -0.5x + 0.75$ **12.** $f(x) = 5x - 7$

13. ERROR ANALYSIS *Describe* and correct the error made in finding the zero of the function $y = 2.3x - 2$.

$y = 2.3(0) - 2$
$y = -2$

14. ★ **MULTIPLE CHOICE** Given the function $y = 12.6x + 3$, for what x-value does $y = 66$?

Ⓐ 0.2 Ⓑ 5 Ⓒ 5.5 Ⓓ 78.6

15. ERROR ANALYSIS *Describe* and correct the error in finding an equation of the best-fitting line using a graphing calculator.

```
LinReg
 y=ax+b
 a=4.47
 b=23.1
 r2=.9989451055
 r=.9994724136
```

Equation of the best-fitting line is $y = 23.1x + 4.47$.

16. ★ **OPEN-ENDED** Give an example of a real-life situation in which you can use linear interpolation to find the zero of a function. *Explain* what the zero means in this situation.

17. CHALLENGE A quantity increases rapidly for 10 years. During the next 10 years, the quantity decreases rapidly.

a. Can you fit a line to the data? *Explain*.

b. How could you model the data using more than one line? *Explain* the steps you could take.

PROBLEM SOLVING

EXAMPLE 1
on p. 335
for Ex. 18

18. SAILBOATS Your school's sailing club wants to buy a sailboat. The table shows the lengths and costs of sailboats.

Length (feet)	11	12	14	14	16	22	23
Cost (dollars)	600	500	1900	1700	3500	6500	6000

a. Make a scatter plot of the data. Let x represent the length of the sailboat. Let y represent the cost of the sailboat.

b. Find an equation that models the cost (in dollars) of a sailboat as a function of its length (in feet).

c. Approximate the cost of a sailboat that is 20 feet long.

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EXAMPLE 2
on p. 336
for Ex. 19

19. **FARMING** The table shows the living space recommended for pigs of certain weights.

Weight (pounds)	40	60	80	100	120	150	230
Area (square feet)	2.5	3	3.5	4	5	6	8

a. Make a scatter plot of the data.

b. Write an equation that models the recommended living space (in square feet) as a function of a pig's weight (in pounds).

c. About how much living space is recommended for a pig weighing 250 pounds?

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EXAMPLE 3
on p. 338
for Ex. 20

20. **TELEVISION STATIONS** The table shows the number of UHF and VHF broadcast television stations each year from 1996 to 2002.

Year	1996	1997	1998	1999	2000	2001	2002
Television stations	1551	1563	1583	1616	1730	1686	1714

a. Find an equation that models the number of broadcast television stations as a function of the number of years since 1996.

b. At approximately what rate did the number of television stations change from 1996 to 2002?

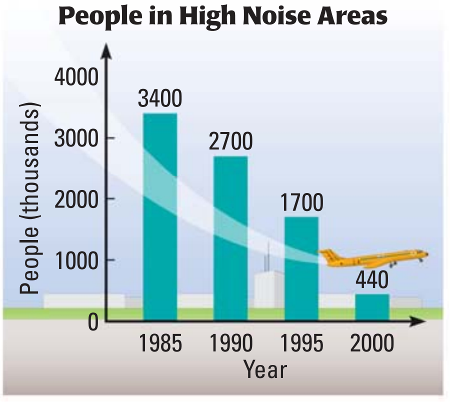
c. Approximate the year in which there were 1750 television stations.

EXAMPLE 4
on p. 338
for Exs. 21–22

21. ★ **SHORT RESPONSE** The table shows the number of people who lived in high noise areas near U.S. airports for several years during the period 1985–2000.

a. Find an equation that models the number of people (in thousands) living in high noise areas as a function of the number of years since 1985.

b. Find the zero of the function from part (a). *Explain* what the zero means in this situation. Is this reasonable?



22. **MULTIPLE REPRESENTATIONS** The table shows the number of U.S. households with personal computers (PCs) from 1994 to 2002.

Year	1994	1995	1996	1997	1998	1999	2000	2001	2002
Households with PCs (millions)	32.0	33.6	38.8	44.0	51.2	61.1	66.0	69.1	72.7

a. **Drawing a Graph** Make a scatter plot of the data in the table.

b. **Writing an Equation** Find an equation that models the number of households with personal computers (in millions) as a function of the number of years since 1994.

c. **Describing in Words** Find the zero of the function from part (b). *Explain* what the zero means in this situation.

= WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE = MULTIPLE REPRESENTATIONS

23. **CHALLENGE** The table shows the estimated populations of mallard ducks and all ducks in North America for several years during the period 1975–2000.

Year	1975	1980	1985	1990	1995	2000
Mallards (thousands)	7727	7707	4961	5452	8269	9470
All ducks (thousands)	37,790	36,220	25,640	25,080	35,870	41,840

a. Make two scatter plots where x is the number of years since 1975 and y is the number of mallards (in thousands) for one scatter plot, while y is the number of ducks (in thousands) for the other scatter plot. *Describe* the correlation of the data in each scatter plot.

b. Can you use the mallard duck population to predict the total duck population? *Explain*.

MIXED REVIEW

Find the sum, difference, product, or quotient.

24. $-19 + (-8)$ *(p. 74)*

25. $-7.3 + 5$ *(p. 74)*

26. $-4.03 + (-3.57)$ *(p. 74)*

27. $-2.8 - (-2.3)$ *(p. 80)*

28. $-4(5)(-5.5)$ *(p. 88)*

29. $-25 \div (-5)$ *(p. 103)*

PREVIEW
Prepare for Lesson 6.1 in Exs. 30–32.

Solve the equation. Check your solution.

30. $x - (-9) = 8$ *(p. 134)*

31. $3x - 4 = -4$ *(p. 141)*

32. $4x + 10x = 98$ *(p. 148)*

QUIZ *for Lessons 5.5–5.7*

1. **PARALLEL LINES** Write an equation of the line that passes through $(-6, 8)$ and is parallel to the line $y = 3x - 15$. *(p. 319)*

PERPENDICULAR LINES **Write an equation of the line that passes through the given point and is perpendicular to the given line.** *(p. 319)*

2. $(5, 5)$, $y = -x + 2$

3. $(10, -3)$, $y = 2x + 24$

4. $(2, 3)$, $x + 2y = -7$

5. **CASSETTE TAPES** The table shows the number of audio cassette tapes shipped for several years during the period 1994–2002. *(pp. 325, 335)*

Year	1994	1996	1998	2000	2002
Tapes shipped (millions)	345	225	159	76	31

a. Write an equation that models the number of tapes shipped (in millions) as a function of the number of years since 1994.

b. At about what rate did the number of tapes shipped change over time?

c. Approximate the year in which 125 million tapes were shipped.

d. Find the zero of the function from part (a). *Explain* what the zero means in this situation.

Internet **ACTIVITY** *Use after Lesson 5.7*

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Keystrokes

5.7 Model Data from the Internet

QUESTION How can you find reliable data on the Internet and use it to predict the total U.S voting-age population in 2010?

EXAMPLE 1 Collect and analyze data

Find data for the total U.S. voting-age population over several years. Use an equation that models the data to predict the total U.S. voting-age population in 2010.

STEP 1 *Find a data source*

Reliable data about the U.S. population can be found in the online *Statistical Abstract*. Go to the address shown below. Click on a link to the most recent version of the *Statistical Abstract*.

Address http://www.census.gov

Voting-Age Population

Year	Total (mil.)
1980	157.1
1988	178.1
1990	182.1
1994	190.3
1996	193.7
1998	198.2

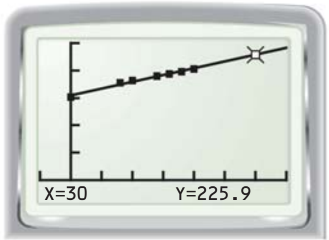
STEP 2 *Find an appropriate data set*

Choose the most recent "Elections" document. In this document, find the table of data entitled "Voting-Age Population."

STEP 3 *Find a model*

Use a graphing calculator to make a scatter plot. Let x represent the number of years since 1980. Let y represent the total U.S. voting-age population (in millions). Find an equation that models the total U.S. voting-age population (in millions) as a function of the number of years since 1980.

▶ $y = 2.23x + 159$



STEP 4 *Predict*

Use the model to predict the total voting-age population in 2010. You can either evaluate the equation for $x = 30$ or trace the graph of the equation, as shown.

▶ The total U.S. voting-age population will be about 225.9 million in 2010.

DRAW CONCLUSIONS

1. In the online *Statistical Abstract*, find data for the total value of agricultural imports over several years beginning with 1990.
2. Make a scatter plot of the data you found in Exercise 1. Find an equation that models the total value of agricultural imports (in millions of dollars) as a function of the number of years since 1990.
3. Predict the year in which the total value of agricultural imports will be \$45,000 million. *Describe* the method you used.

MIXED REVIEW *of Problem Solving*

Lessons 5.5–5.7

1. **MULTI-STEP PROBLEM** The table shows the value of primary and secondary schools built in the U.S. each year from 1995 to 2000.

Year	Value (millions of dollars)
1995	1245
1996	1560
1997	2032
1998	2174
1999	2420
2000	2948

   a. Make a scatter plot of the data.

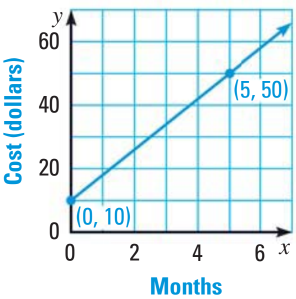
   b. Write an equation that models the value (in millions of dollars) of the schools built as a function of the number of years since 1995.

   c. At approximately what rate did the value change from 1995 to 2000?

   d. In what year would you predict the value of the schools built in the U.S. to be $3,600,000,000?

2. **GRIDDED ANSWER** A map of a city shows streets as lines on a coordinate grid. State Street has a slope of $-\frac{1}{2}$. Park Street runs perpendicular to State Street. What is the slope of Park Street on the map?

3. **OPEN-ENDED** The graph represents the cost for one kayak owner for storing a kayak at a marina over time. The total cost includes a standard initial fee and a monthly storage fee. Suppose a different kayak owner pays a lower initial fee during a special promotion. Write an equation that could give the total cost as a function of the number of months of storage for this kayak owner.



4. **SHORT RESPONSE** The table shows the heights and corresponding lengths of horses in a stable. Make a scatter plot of the data. *Describe* the correlation of the data.

Height (hands)	Length (inches)
17.0	76
16.0	72
16.2	74
15.3	71
15.1	69
16.3	75

5. **EXTENDED RESPONSE** The table shows the percent of revenue from U.S. music sales made through music clubs from 1998 through 2003.

Year	Percent of revenue
1998	9
1999	7.9
2000	7.6
2001	6.1
2002	4
2003	4.1

   a. Find an equation that models the percent of revenue from music clubs as a function of the number of years since 1998.

   b. At approximately what rate did the percent of revenue from music clubs change from 1998 to 2003?

   c. Find the zero of the function. *Explain* what the zero means in this situation.

6. **SHORT RESPONSE** The cost of bowling includes a $4.00 fee per game and a shoe rental fee. Shoes for adults cost $2.25. Shoes for children cost $1.75. Write equations that give the total cost of bowling for an adult and for a child as functions of the number of games bowled. How are the graphs of the equations related? *Explain*.

5 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Writing Linear Equations in a Variety of Forms

Using given information about a line, you can write an equation of the line in three different forms.

Form	Equation	Important information
Slope-intercept form	$y = mx + b$	• The slope of the line is m. • The y-intercept of the line is b.
Point-slope form	$y - y_1 = m(x - x_1)$	• The slope of the line is m. • The line passes through (x_1, y_1).
Standard form	$Ax + By = C$	• A, B, and C are real numbers. • A and B are not both zero.

Big Idea 2

Using Linear Models to Solve Problems

You can write a linear equation that models a situation involving a constant rate of change. Analyzing given information helps you choose a linear model.

Choosing a Linear Model	
If this is what you know . . .	**. . . then use this equation form**
constant rate of change and initial value	slope-intercept form
constant rate of change and one data pair	slope-intercept form or point-slope form
two data pairs and the fact that the rate of change is constant	slope-intercept form or point-slope form
the sum of two variable quantities is constant	standard form

Big Idea 3

Modeling Data with a Line of Fit

You can use a line of fit to model data that have a positive or negative correlation. The line or an equation of the line can be used to make predictions.

Step 1 Make a scatter plot of the data.

Step 2 Decide whether the data can be modeled by a line.

Step 3 Draw a line that appears to follow the trend in data closely.

Step 4 Write an equation using two points on the line.

Step 5 Interpolate (between known values) or extrapolate (beyond known values) using the line or its equation.

5 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- point-slope form, *p. 302*
- converse, *p. 319*
- perpendicular, *p. 320*
- scatter plot, *p. 325*
- positive correlation, negative correlation, relatively no correlation, *p. 325*
- line of fit, *p. 326*
- best-fitting line, *p. 335*
- linear regression, *p. 335*
- interpolation, *p. 335*
- extrapolation, *p. 336*
- zero of a function, *p. 337*

VOCABULARY EXERCISES

1. Copy and complete: If a best-fitting line falls from left to right, then the data have a(n) __?__ correlation.

2. Copy and complete: Using a linear function to approximate a value beyond a range of known values is called __?__.

3. **WRITING** What is the zero of a function, and how does it relate to the function's graph? *Explain.*

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 5.

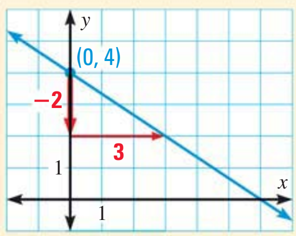
5.1 Write Linear Equations in Slope-Intercept Form *pp. 283–289*

EXAMPLE

Write an equation of the line shown.

$y = mx + b$ **Write slope-intercept form.**

$y = -\frac{2}{3}x + 4$ **Substitute $-\frac{2}{3}$ for m and 4 for b.**



EXERCISES

EXAMPLES 1 and 5 on pp. 283, 285 for Exs. 4–7

Write an equation in slope-intercept form of the line with the given slope and y-intercept.

4. slope: 3
y-intercept: −10

5. slope: $\frac{4}{9}$
y-intercept: 5

6. slope: $-\frac{2}{11}$
y-intercept: 7

7. **GIFT CARD** You have a \$25 gift card for a bagel shop. A bagel costs \$1.25. Write an equation that gives the amount (in dollars) that remains on the card as a function of the total number of bagels you have purchased so far. How much money is on the card after you buy 2 bagels?

5 CHAPTER REVIEW

5.2 Use Linear Equations in Slope-Intercept Form *pp. 292–299*

EXAMPLE

Write an equation of the line that passes through the point (−2, −6) and has a slope of 2.

STEP 1 **Find** the y-intercept.

$y = mx + b$ — Write slope-intercept form.

$-6 = 2(-2) + b$ — Substitute 2 for m, −2 for x, and −6 for y.

$-2 = b$ — Solve for b.

STEP 2 **Write** an equation of the line.

$y = mx + b$ — Write slope intercept form.

$y = 2x - 2$ — Substitute 2 for m and −2 for b.

EXERCISES

EXAMPLE 1 on p. 292 for Exs. 8–10

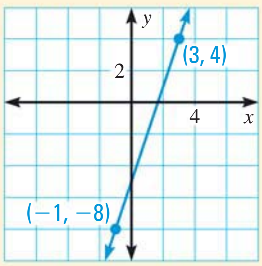
Write an equation in slope-intercept form of the line that passes through the given point and has the given slope m.

8. $(-3, -1)$; $m = 4$ **9.** $(-2, 1)$; $m = 1$ **10.** $(8, -4)$; $m = -3$

5.3 Write Linear Equations in Point-Slope Form *pp. 302–308*

EXAMPLE

Write an equation in point-slope form of the line shown.



STEP 1 **Find** the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - 4}{-1 - 3} = \frac{-12}{-4} = 3$$

STEP 2 **Write** an equation. Use (3, 4).

$y - y_1 = m(x - x_1)$ — Write point-slope form.

$y - 4 = 3(x - 3)$ — Substitute 3 for m, 3 for x_1, and 4 for y_1.

EXERCISES

EXAMPLES 3 and 5 on pp. 303, 304 for Exs. 11–14

Write an equation in point-slope form of the line that passes through the given points.

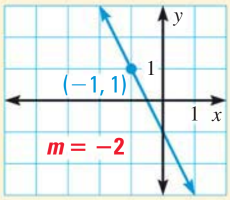
11. (4, 7), (5, 1) **12.** (9, −2), (−3, 2) **13.** (8, −8), (−3, −2)

14. BUS TRIP A bus leaves at 10 A.M. to take students on a field trip to a historic site. At 10:25 A.M., the bus is 100 miles from the site. At 11:15 A.M., the bus is 65 miles from the site. The bus travels at a constant speed. Write an equation in point-slope form that relates the distance (in miles) from the site and the time (in minutes) after 10:00 A.M. How far is the bus from the site at 11:30 A.M.?

5.4 Write Linear Equations in Standard Form *pp. 311–316*

EXAMPLE

Write an equation in standard form of the line shown.



$y - y_1 = m(x - x_1)$ — **Write point-slope form.**

$y - 1 = -2(x - (-1))$ — **Substitute 1 for y_1, −2 for m, and −1 for x_1.**

$y - 1 = -2x - 2$ — **Distributive property**

$2x + y = -1$ — **Collect variable terms on one side, constants on the other.**

EXERCISES

EXAMPLES 2 and 5 on pp. 311, 313 for Exs. 15–17

Write an equation in standard form of the line that has the given characteristics.

15. Slope: −4; passes through (−2, 7)

16. Passes through (−1, −5) and (3, 7)

17. COSTUMES You are buying ribbon to make costumes for a school play. Organza ribbon costs $.07 per yard. Satin ribbon costs $.04 per yard. Write an equation to model the possible combinations of yards of organza ribbon and yards of satin ribbon you can buy for $5. List several possible combinations.

5.5 Write Equations of Parallel and Perpendicular Lines *pp. 319–324*

EXAMPLE

Write an equation of the line that passes through (−4, −2) and is perpendicular to the line $y = 4x - 7$.

The slope of the line $y = 4x - 7$ is 4. The slope of the perpendicular line through (−4, −2) is $-\frac{1}{4}$. Find the y-intercept of the perpendicular line.

$y = mx + b$ — **Write slope-intercept form.**

$-2 = -\frac{1}{4}(-4) + b$ — **Substitute $-\frac{1}{4}$ for m, −4 for x, and −2 for y.**

$-3 = b$ — **Solve for b.**

An equation of the perpendicular line through (−4, −2) is $y = -\frac{1}{4}x - 3$.

EXERCISES

EXAMPLES 1 and 4 on pp. 319, 321 for Exs. 18–20

Write an equation of the line that passes through the given point and is (a) parallel to the given line and (b) perpendicular to the given line.

18. $(0, 2), y = -4x + 6$

19. $(2, -3), y = -2x - 3$

20. $(6, 0), y = \frac{3}{4}x - \frac{1}{4}$

5 CHAPTER REVIEW

5.6 Fit a Line to Data *pp. 323–331*

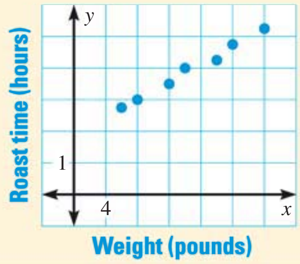
EXAMPLE

The table shows the time needed to roast turkeys of different weights. Make a scatter plot of the data. *Describe* the correlation of the data.

Weight (pounds)	6	8	12	14	18	20	24
Roast time (hours)	2.75	3.00	3.50	4.00	4.25	4.75	5.25

Treat the data as ordered pairs. Let x represent the turkey weight (in pounds), and let y represent the time (in hours) it takes to roast the turkey. Plot the ordered pairs as points in a coordinate plane.

The scatter plot shows a positive correlation, which means that heavier turkeys tend to require more time to roast.



EXERCISES

EXAMPLE 2 on p. 326 for Ex. 21

21. AIRPORTS The table shows the number of airports in the Unites States for several years during the period 1990–2001. Make a scatter plot of the data. *Describe* the correlation of the data.

Years	1990	1995	1998	1999	2000	2001
Airports (thousands)	17.5	18.2	18.8	19.1	19.3	19.3

5.7 Predict with Linear Models *pp. 335–341*

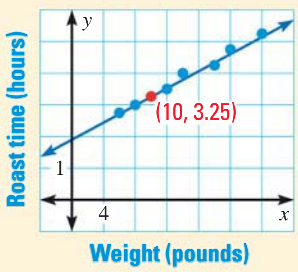
EXAMPLE

Use the scatter plot from the example for Lesson 5.6 above to estimate the time (in hours) it takes to roast a 10 pound turkey.

Draw a line that appears to fit the points in the scatter plot closely. There should be approximately as many points above the line as below it.

Find the point on the line whose x-coordinate is 10. At that point, you can see that the y-coordinate is about 3.25.

▶ It takes about 3.25 hours to roast a 10 pound turkey.



EXERCISES

EXAMPLE 2 on p. 336 for Ex. 22

22. COOKING TIMES Use the graph in the Example above to estimate the time (in hours) it takes to roast a turkey that weighs 30 pounds. *Explain* how you found your answer.

5 CHAPTER TEST

Write an equation in slope-intercept form of the line with the given slope and y-intercept.

1. slope: 5
y-intercept: -7

2. slope: $\frac{2}{5}$
y-intercept: -2

3. slope: $-\frac{4}{3}$
y-intercept: 1

Write an equation in slope-intercept form of the line that passes through the given point and has the given slope m.

4. $(-2, -8)$; $m = 3$

5. $(1, 1)$; $m = -4$

6. $(-1, 3)$; $m = -6$

Write an equation in point-slope form of the line that passes through the given points.

7. $(4, 5)$, $(2, 9)$

8. $(-2, 2)$, $(8, -3)$

9. $(3, 4)$, $(1, -6)$

Write an equation in standard form of the line with the given characteristics.

10. Slope: 10; passes through $(6, 2)$

11. Passes through $(-3, 2)$ and $(6, -1)$

Write an equation of the line that passes through the given point and is (a) parallel to the given line and (b) perpendicular to the given line.

12. $(2, 0)$, $y = -5x + 3$

13. $(-1, 4)$, $y = -x - 4$

14. $(4, -9)$, $y = \frac{1}{4}x + 2$

Make a scatter plot of the data. Draw a line of fit. Write an equation of the line.

15.

x	0	1	2	3	4
y	15	35	53	74	94

16.

x	0	2	4	8	10
y	-2	6	15	38	50

17. FIELD TRIP Your science class is taking a field trip to an observatory. The cost of a presentation and a tour of the telescope is \$60 for the group plus an additional \$3 per person. Write an equation that gives the total cost C as a function of the number of people p in the group.

18. GOLF FACILITIES The table shows the number of golf facilities in the United States during the period 1997–2001.

a. Make a scatter plot of the data where x is the number of years since 1997 and y is the number of golf facilities (in thousands).

b. Write an equation that models the number of golf facilities (in thousands) as a function of the number of years since 1997.

c. At about what rate did the number of golf facilities change during the period 1997–2001?

d. Use the equation from part (b) to predict the number of golf facilities in 2004.

e. Predict the year in which the number of golf facilities reached 16,000. *Explain* how you found your answer.

Year	Golf facilities (thousands)
1997	14.6
1998	14.9
1999	15.2
2000	15.5
2001	15.7

5 ★ Standardized TEST PREPARATION

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,
  or
- solution is without error but incomplete

No Credit
- no solution is given,
  or
- solution makes no sense

SHORT RESPONSE QUESTIONS

PROBLEM

The average monthly cost of basic cable increased by about \$1.47 each year from 1986 to 2003. In 1986 the average monthly cost of basic cable was \$10.67. Write an equation that gives the monthly cost (in dollars) of basic cable as a function of the number of years since 1986. In what year was the monthly cost of basic cable \$31.25? *Explain* your reasoning.

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

A verbal model shows how the equation is obtained.

Let y be the average monthly cost x years since 1986.

Monthly cost	=	Cost in 1986	+	Cost increase per year	•	Years since 1986
y	=	**10.67**	+	**1.47**	•	x

To find the year when the monthly cost was \$31.25, substitute 31.25 for y and solve for x.

Calculations are performed correctly.

$$y = 10.67 + 1.47x$$
$$31.25 = 10.67 + 1.47x$$
$$14 = x$$

The question is answered correctly.

The monthly cost was \$31.25 fourteen years after 1986, or in 2000.

SAMPLE 2: Partial credit solution

The equation is correct, and the student has explained what the variables represent.

Let y be the monthly cost. Let x be the number of years since 1986.

Monthly cost	=	Cost in 1986	+	Cost increase per year	•	Years since 1986
y	=	**10.67**	+	**1.47**	•	x

The answer is incorrect, because the student mistakenly substituted the cost for the variable that represents the years since 1986.

To find the year when the cost was \$31.25, substitute 31.25 for x.

$$y = 10.67 + 1.47x$$
$$= 10.67 + 1.47(31.25) \approx 56.61$$

The cost was \$31.25 about 57 years after 1986, or in 2042.

SAMPLE 3: Partial credit solution

The equation and answer are correct. There are no explanations to support the student's work.

$$y = 10.67 + 1.47x$$
$$31.25 = 10.67 + 1.47x$$
$$14 = x$$

The monthly cost was $31.25 fourteen years after 1986, or in 2000.

SAMPLE 4: No credit solution

The student's reasoning is incorrect, and the equation is incorrect. The answer is incorrect.

Year when cost is $31.25 = $31.25 ÷ 1.47

$$y = 31.25 \div 1.47 \approx 21.25$$

The year is about 21 years after 1986, or in 2007.

PRACTICE Apply the Scoring Rubric

Score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning.

PROBLEM A hot air balloon is flying at an altitude of 870 feet. It descends at a rate of 15 feet per minute. Write an equation that gives the altitude (in feet) of the balloon as a function of the time (in minutes) since it began its descent. Find the time it takes the balloon to reach an altitude of 12 feet. *Explain* your reasoning.

1. Let y be the altitude (in feet) after x minutes.

$\boxed{\text{Final altitude}} = \boxed{\text{Starting altitude}} + \boxed{\text{Decrease in altitude per minute}} \cdot \boxed{\text{Minutes}}$

$$y = 870 + (-15)x = 870 - 15(12) = 690$$

The balloon will take 690 minutes to reach an altitude of 12 feet.

2. Let y be the altitude (in feet) after x minutes.

$\boxed{\text{Final altitude}} = \boxed{\text{Starting altitude}} + \boxed{\text{Decrease in altitude per minute}} \cdot \boxed{\text{Minutes}}$

$$y = 870 + (-15)x$$
$$12 = 870 - 15x$$
$$57.2 = x$$

The balloon will take about 57 minutes to reach an altitude of 12 feet.

3. $870 \div 15 = 58$

The balloon will take 58 minutes to reach an altitude of 12 feet.

5 ★ Standardized TEST PRACTICE

SHORT RESPONSE

1. You have \$50 to spend on pretzels and juice drinks for a school dance. A box of pretzels costs \$3.50, and a package of juice drinks costs \$5.00. Write an equation in standard form that models the possible combinations of boxes of pretzels and packages of juice drinks that you can buy. What is the greatest number of boxes of pretzels you can buy? *Explain*.

2. You and your family are traveling home in a car at an average speed of 60 miles per hour. At noon you are 180 miles from home.
   a. Write an equation that gives your distance from home (in miles) as a function of the number of hours since noon.
   b. *Explain* why the graph of this equation is a line with a negative slope.

3. Robyn needs \$2.20 to buy a bag of trail mix. Write an equation in standard form that models the possible combinations of nickels and dimes she could use to pay for the mix. How many nickels would she need if she used 14 dimes? *Explain* your reasoning.

4. On a street map, Main Street and Maple Street can be modeled by the equations $y = ax + 6$ and $x + 2y = 4$. For what value of a are the streets parallel? For what value of a are the streets perpendicular? *Justify* your answers.

5. The table shows the projected dollar amount spent per person in the U.S. on interactive television for several years during the period 1998–2006.

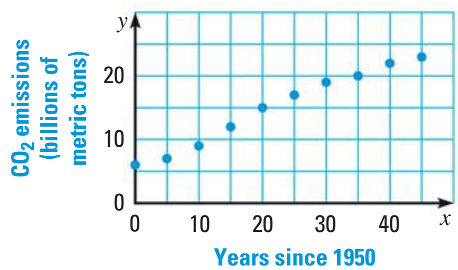
Year	Spending per person (dollars)
1998	0
2000	2.86
2002	6.63
2004	9.50
2006	12.85

   a. Make a scatter plot of the data.
   b. Predict the year in which spending per person in the U.S. on interactive television will reach \$20. *Explain* how you found your answer.

6. You are mountain biking on a 10 mile trail. You biked 4 miles before stopping to take a break. After your break, you bike at a rate of 9 miles per hour.
   a. Write an equation that gives the length (in miles) of the trail you have completed as a function of the number of hours since you began.
   b. How much time (in hours and minutes) after your break will it take you to complete the entire trail? *Explain*.

7. A guide gives dogsled tours during the winter months. The guide charges one amount for the first hour of a tour and a different amount for each hour after the first. You paid \$55 for a 2 hour dogsled tour. Your friend paid \$70 for a 3 hour tour.
   a. *Explain* why this situation can be modeled by a linear equation.
   b. Write an equation that gives the cost (in dollars) of a dogsled tour as a function of the number of hours after the first hour of the tour.

8. The scatter plot shows the total carbon dioxide emissions throughout the world for several years during the period 1950–1995.



   a. Write an equation that models the carbon dioxide emissions (in billions of metric tons) as a function of the number of years since 1950.
   b. At about what rate did the amount of carbon dioxide emissions increase from 1950 to 1995? *Explain* how you found your answer.

STATE TEST PRACTICE
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MULTIPLE CHOICE

9. Which equation represents the line that passes through (0, 8) and (2, 0)?

(A) $y = 4x + 2$ (B) $y = -4x + 2$

(C) $y = 4x + 8$ (D) $y = -4x + 8$

10. Which equation represents the line with a slope of 5 and a y-intercept of 2?

(A) $y = 2x + 5$ (B) $y = 2x - 5$

(C) $y = 5x + 2$ (D) $y = 5x - 2$

11. Which function has the values $f(1) = 8$ and $f(7) = -10$?

(A) $f(x) = -3x + 11$ (B) $f(x) = -2x + 10$

(C) $f(x) = -3x + 25$ (D) $f(x) = 3x - 24$

GRIDDED ANSWER

12. What is the slope of a line that is perpendicular to the line $y = -2x - 7$?

13. What is the y-intercept of the line that has a slope of $\frac{1}{2}$ and passes through (5, 4)?

14. What is the y-intercept of the line that is parallel to the line $y = 2x - 3$ and passes through the point (6, 11)?

15. What is the zero of the function $f(x) = -\frac{4}{5}x + 9$?

16. The graph of the equation $Ax + y = 2$ is a line that passes through (−2, 8). What is the value of A?

EXTENDED RESPONSE

17. The table shows the time several students spent studying for an exam and each student's grade on the exam.

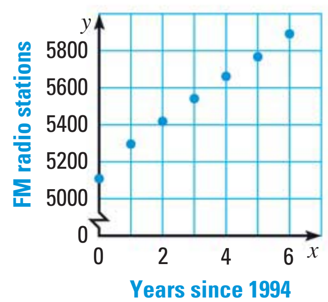
Study time (hours)	1.5	0.5	0.5	1	1	3	2.5	3	0
Grade	90	60	70	72	80	88	89	94	58

a. Make a scatter plot of the data.

b. Write an equation that models a student's exam grade as a function of the time (in hours) the student spent studying for the exam.

c. How many hours would you need to study in order to earn a grade of 93 on the exam? *Justify* your answer using the data above.

18. The scatter plot shows the number of FM radio stations in the United States for several years during the period 1994–2000.



a. *Describe* the correlation of the data.

b. Write an equation that models the number of FM radio stations in the United States as a function of the number of years since 1994.

c. At about what rate did the number of radio stations change during the period 1994–2000?

d. Find the zero of the function from part (b). *Explain* what the zero means in this situation.