13 Probability and Data Analysis

Before

In previous courses, you learned the following skills, which you'll use in Chapter 13: finding the mean, median, and mode(s) of data and using a display to analyze data.

Prerequisite Skills

VOCABULARY CHECK

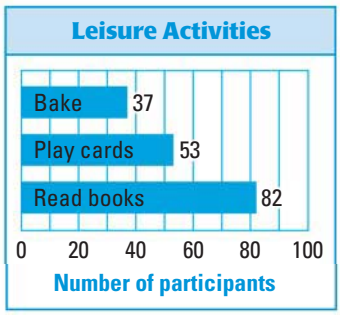
1. Copy and complete: The __?__ of a numerical data set is the middle number when the values are written in numerical order.

SKILLS CHECK

Find the mean, median, and mode(s) of the data. *(Review p. 918 for 13.6.)*

2. 0.2, 1.3, 0.9, 1.5, 2.1, 1.8, 0.6
3. 103, 121, 111, 194, 99, 160, 134, 160

In Exercises 4 and 5, use the bar graph, which shows the numbers of adults who participate in leisure activities, according to the results of a survey. *(Review p. 933 for 13.7.)*



4. Which activities have fewer than 60 participants?
5. Of those surveyed, how many more read books than play cards?

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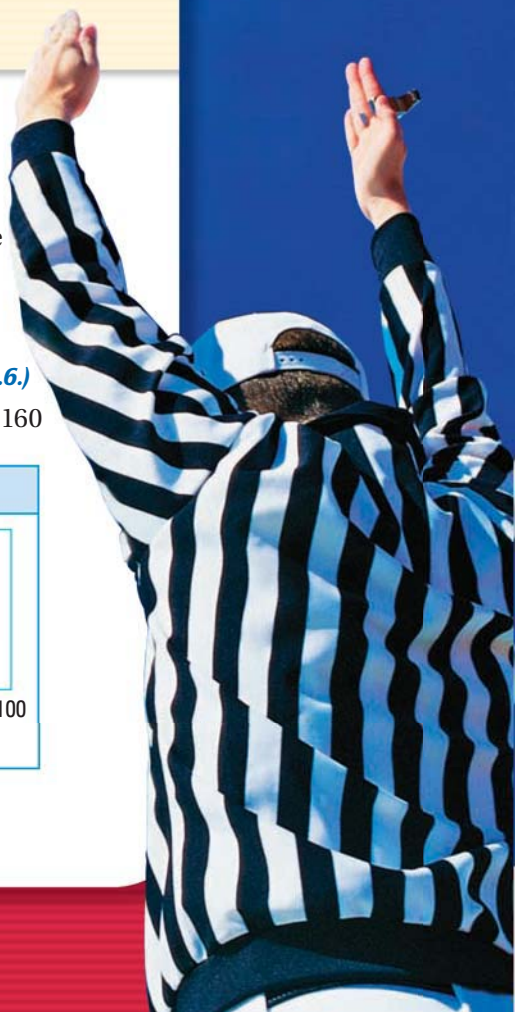


Now

In Chapter 13, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 895. You will also use the key vocabulary listed below.

Big Ideas

1. **Finding probabilities of simple and compound events**
2. **Analyzing sets of data**
3. **Making and interpreting data displays**

KEY VOCABULARY

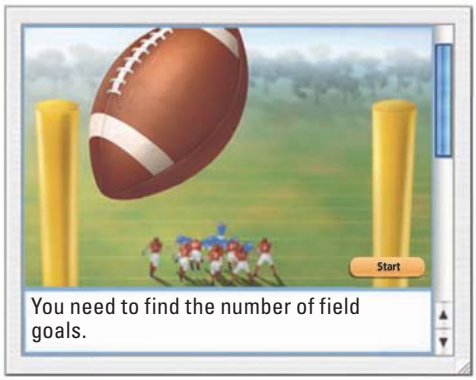
- outcome, *p. 843*
- event, *p. 843*
- probability, *p. 843*
- odds, *p. 845*
- permutation, *p. 851*
- combination, *p. 856*
- compound event, *p. 861*
- survey, *p. 871*
- sample, *p. 871*
- measure of dispersion, *p. 876*
- range, *p. 876*
- stem-and-leaf plot, *p. 881*
- frequency, *p. 882*
- histogram, *p. 882*
- box-and-whisker plot, *p. 887*
- interquartile range, *p. 888*
- outlier, *p. 889*

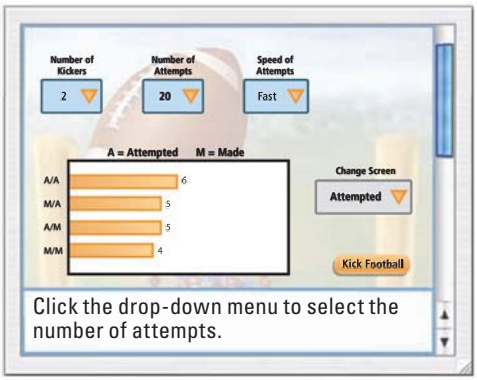
Why?

You can use probability and data analysis to make predictions. For example, you can use data about a kicker's past successes in football games to find the chance of his success in the future.

Animated Algebra

The animation illustrated below for Exercise 22 on page 848 helps you to answer this question: What is the probability that the kicker makes an attempted field goal?





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Other animations for Chapter 13: pages 845, 856, 875, and 887

13.1 Find a Probability

MATERIALS • paper bag

QUESTION **What is the chance that you would select the initials of a student in your class from a bag of letters?**

You can perform an experiment and record the results to approximate the likelihood of selecting the initials of a student in your class.

EXPLORE Perform an experiment

STEP 1 Select letters

Write each of the 26 letters of the alphabet on separate pieces of paper. Put all of the letters into a bag. Select a letter at random (without looking into the bag). Replace the letter and select a second letter at random.

STEP 2 Record the results

Record the results of the selections in a table like the one shown.

- If the first letter is the first initial of any student in your class, put a tally mark in the "first initial" column.
- If the second letter is the last initial of any student in your class, put a tally mark in the "last initial" column.
- If the two letters are the first and last initials of any student in your class, put a tally mark in the "both initials" column, but do not put a tally mark in the other columns.

Perform this experiment 30 times.

	First initial	Last initial	Both initials
Tally	卌	卌 II	I
Frequency	?	?	?

STEP 3 Record the frequencies

Record the *frequency*, the total number of tally marks, of each possible result.

DRAW CONCLUSIONS Use your observations to complete these excercises

1. For what fraction of the times that you performed the experiment did you select the first initial of a student in your class? the last initial? both?
2. Which of these results do you think is least likely to happen if you repeat the experiment 30 more times? *Explain* your choice.
3. **REASONING** You perform the experiment 90 times. How many times do you expect to select both the first and last initials of a student in your class? *Explain* how you made your prediction.

13.1 Find Probabilities and Odds

Before You made organized lists and tree diagrams.

Now You will find sample spaces and probabilities.

Why? So you can find the likelihood of an event, as in Example 2.

Key Vocabulary
- **outcome**
- **event**
- **sample space**
- **probability**
- **odds**

A possible result of an experiment is an **outcome**. For instance, when you roll a number cube there are 6 possible outcomes: a 1, 2, 3, 4, 5, or 6. An **event** is an outcome or a collection of outcomes, such as rolling an odd number. The set of all possible outcomes is called a **sample space**.

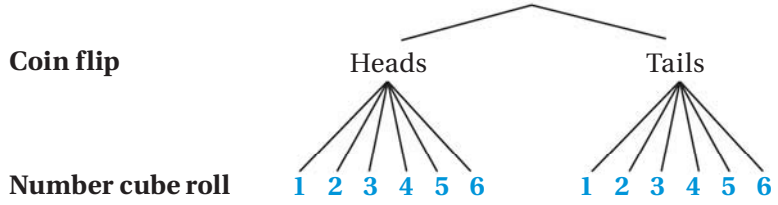
EXAMPLE 1 Find a sample space

You flip a coin and roll a number cube. How many possible outcomes are in the sample space? List the possible outcomes.

REVIEW TREE DIAGRAMS
For help with tree diagrams, see p. 931.

Solution

Use a tree diagram to find the outcomes in the sample space.



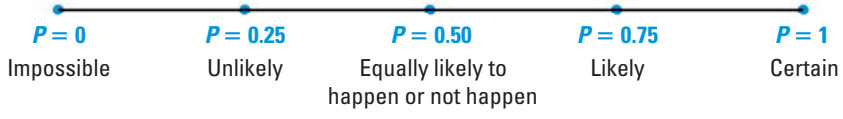
The sample space has 12 possible outcomes. They are listed below.

Heads, 1 Heads, 2 Heads, 3 Heads, 4 Heads, 5 Heads, 6
Tails, 1 Tails, 2 Tails, 3 Tails, 4 Tails, 5 Tails, 6

✓ GUIDED PRACTICE for Example 1

1. You flip 2 coins and roll a number cube. How many possible outcomes are in the sample space? List the possible outcomes.

PROBABILITY The **probability of an event** is a measure of the likelihood, or chance, that the event will occur. Probability is a number from 0 to 1 and can be expressed as a decimal, fraction, or percent.



THEORETICAL PROBABILITY The outcomes for a specified event are called *favorable outcomes*. When all outcomes are equally likely, the **theoretical probability** of the event can be found using the following:

$$\textbf{Theoretical probability} = \frac{\textbf{Number of favorable outcomes}}{\textbf{Total number of outcomes}}$$

The probability of event A is written as $P(A)$.

EXAMPLE 2 Find a theoretical probability

T-SHIRTS You and your friends designed T-shirts with silk screened emblems, and you are selling the T-shirts to raise money. The table below shows the number of T-shirts you have in each design. A student chooses a T-shirt at random. What is the probability that the student chooses a red T-shirt?

	Gold emblem	Silver emblem
Green T-shirt	10	8
Red T-shirt	6	6

Solution

You and your friends have a total of $10 + 6 + 8 + 6 = 30$ T-shirts. So, there are 30 possible outcomes. Of all the T-shirts, 12 T-shirts are red. There are 12 favorable outcomes.

$$P(\text{red T-shirt}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{\text{Number of red T-shirts}}{\text{Total number of T-shirts}}$$

$$= \frac{12}{30}$$

$$= \frac{2}{5}$$

✓ GUIDED PRACTICE for Example 2

2. **T-SHIRTS** In Example 2, what is the probability that the student chooses a T-shirt with a gold emblem?
3. You toss a coin and roll a number cube. What is the probability that the coin shows tails and the number cube shows 4?

EXPERIMENTAL PROBABILITY An **experimental probability** is based on repeated *trials* of an experiment. The number of trials is the number of times the experiment is performed. Each trial in which a favorable outcome occurs is called a *success*.

$$\textbf{Experimental probability} = \frac{\textbf{Number of successes}}{\textbf{Number of trials}}$$

EXAMPLE 3 Standardized Test Practice

Each section of the spinner shown has the same area. The spinner was spun 20 times. The table shows the results. For which color is the experimental probability of stopping on the color the same as the theoretical probability?

Spinner Results			
Red	Green	Blue	Yellow
5	9	3	3

Ⓐ Red Ⓑ Green Ⓒ Blue Ⓓ Yellow

Solution

The theoretical probability of stopping on each of the four colors is $\frac{1}{4}$. Use the outcomes in the table to find the experimental probabilities.

$P(\text{red}) = \frac{5}{20} = \frac{1}{4}$ $P(\text{green}) = \frac{9}{20}$ $P(\text{blue}) = \frac{3}{20}$ $P(\text{yellow}) = \frac{3}{20}$

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

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ODDS The odds of an event compare the number of favorable and unfavorable outcomes when all outcomes are equally likely.

$$\textbf{Odds in favor} = \frac{\textbf{Number of favorable outcomes}}{\textbf{Number of unfavorable outcomes}}$$

$$\textbf{Odds against} = \frac{\textbf{Number of unfavorable outcomes}}{\textbf{Number of favorable outcomes}}$$

EXAMPLE 4 Find the odds

READING Odds are read as the ratio of one number to another. For instance, the odds $\frac{3}{1}$ are read as "three to one." Odds are usually written as $a : b$.

SPINNER In Example 3, find the odds against stopping on green.

Solution

The 4 possible outcomes are all equally likely. Green is the 1 favorable outcome. The other 3 colors are unfavorable outcomes.

$$\text{Odds against green} = \frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}} = \frac{3}{1} \text{ or } 3 : 1.$$

✓ GUIDED PRACTICE for Examples 3 and 4

4. In Example 3, for which color is the experimental probability of stopping on the color greater than the theoretical probability?

5. In Example 3, what are the odds in favor of stopping on blue?

13.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3 and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14–16, 21, and 22

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A number that describes the likelihood of an event is the __?__ of the event.

2. ★ **WRITING** *Explain* how the probability of an event differs from the odds in favor of the event when all outcomes are equally likely.

EXAMPLE 1 on p. 843 for Exs. 3–6

SAMPLE SPACE **In Exercises 3–6, find the number of possible outcomes in the sample space. Then list the possible outcomes.**

3. A bag contains 4 red cards numbered 1–4, 4 white cards numbered 1–4, and 4 black cards numbered 1–4. You choose a card at random.

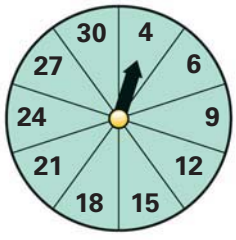
4. You toss two coins.

5. You roll a number cube and toss three coins.

6. You roll two number cubes.

EXAMPLE 2 on p. 844 for Exs. 7–8

PROBABILITY AND ODDS **In Exercises 7–13, refer to the spinner shown. The spinner is divided into sections with the same area.**



7. What is the probability that the spinner stops on a multiple of 3?

8. **ERROR ANALYSIS** *Describe* and correct the error in finding the probability of stopping on a multiple of 9.

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{2}{10} = \frac{1}{5}$$

EXAMPLE 3 on p. 845 for Exs. 9–10

9. You spin the spinner 30 times. It stops on 12 three times. What is the experimental probability of stopping on 12?

10. You spin the spinner 10 times. It stops on an even number 6 times. What is the experimental probability of stopping on an even number?

EXAMPLE 4 on p. 845 for Exs. 11–14

11. What are the odds in favor of stopping on a multiple of 4?

12. What are the odds against stopping on a number less than 12?

13. **ERROR ANALYSIS** *Describe* and correct the error in finding the odds in favor of stopping on a multiple of 3.

$$\text{Odds in favor of a multiple of 3} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{9}{10} \text{ or } 9:10$$

14. ★ **MULTIPLE CHOICE** The odds in favor of an event are 5 : 8. What are the odds against the event?

   Ⓐ 3 : 8 Ⓑ 8 : 3 Ⓒ 5 : 8 Ⓓ 8 : 5

15. ★ **OPEN-ENDED** *Describe* a real-world event whose probability is 0. *Describe* another real-world event whose probability is 1.

16. ★ **MULTIPLE CHOICE** According to a meteorologist, there is a 40% chance that it will rain today. What are the odds in favor of rain?

Ⓐ 2 : 5 Ⓑ 2 : 3 Ⓒ 3 : 2 Ⓓ 4 : 1

17. **NUMBER CUBES** Make a table showing all of the possible sums that result from rolling two number cubes. (Columns represent the possible outcomes of the first number cube. Rows represent the possible outcomes of the second number cube. The cells of the table represent the sums of the two outcomes.) Then find the probability of rolling each sum.

18. **CHALLENGE** A bag holds red, white, and blue marbles. You randomly draw a marble from the bag. The odds against drawing a white marble are 47 : 3.

a. There are fewer than 100 marbles in the bag. How many marbles are in the bag? *Justify* your answer.

b. The probability of drawing a red marble is 0.5. What is the probability of drawing a blue marble? *Explain* how you found your answer.

PROBLEM SOLVING

EXAMPLE 2 on p. 844 for Exs. 19–20

19. **MUSIC PROGRAM** You have created a playlist of 7 songs on your MP3 player. You play these songs in a random shuffle, where each song has an equally likely chance of being played. What is the probability that the second song on the list will be played first?

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20. **SURVEY** A survey asked a total of 600 students (100 male students and 100 female students who were 11, 13, and 15 years old) about their exercise habits. The table shows the numbers of students who said they exercise 2 hours or more each week.

	11 years	13 years	15 years
Female	53	57	51
Male	65	68	67

a. What is the probability that a randomly selected female student who participated in this survey exercises 2 hours or more each week?

b. What is the probability that a randomly selected 15-year-old student who participated in this survey exercises 2 hours or more each week?

c. What is the probability that a randomly selected student who participated in this survey exercises 2 hours or more each week?

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EXAMPLES 2 and 4 on pp. 844–845 for Ex. 21

21. ★ **SHORT RESPONSE** Suppose there are 15 girls and 12 boys in your homeroom. The teacher chooses one student representative at random. What is the probability that a boy is chosen? What are the odds in favor of choosing a boy? *Explain* how the probablity and odds are related.

22. ★ **EXTENDED RESPONSE** The table shows the 2003 regular season field goal statistics for kicker Adam Vinatieri.

	Point difference at end of game		
	0–7 points	8–14 points	≥ 15 points
Field goals attempted	20	11	3
Field goals made	16	7	2

a. During the 2003 regular season, what was the probability that Adam Vinatieri would make an attempted field goal, regardless of the point difference?

b. Find the probabilities that Vinatieri made an attempted field goal when the point difference at the end of the game was 0–7 points, 8–14 points, and at least 15 points.

c. During what kinds of games was Adam Vinatieri most likely to make attempted field goals? *Justify* your answer.

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23. **CHALLENGE** The table shows the results of Congressional elections that involved incumbent candidates (representatives or senators who ran for re-election) during the period 1980–2000.

	Incumbent representatives		Incumbent senators	
	Ran	Re-elected	Ran	Re-elected
Presidential election year	2373	2235	163	130
Midterm election year	1984	1873	145	130

a. Did a representative or a senator have a better chance of being re-elected? *Justify* your answer using the data in the table.

b. Did a member of Congress have a better chance of being re-elected during a presidential election year than during a midterm election year? *Justify* your answer.

MIXED REVIEW

PREVIEW
Prepare for Lesson 13.2 in Exs. 24–26.

Use the indicated counting method to answer the question. *(p. 931)*

24. You have 3 posters to hang beside each other on a wall. In how many different ways can you hang the posters? (Make a list.)

25. Members of a credit union choose a personal identification number (PIN) for their debit card. The PIN consists of 4 digits from 0 to 9. Digits cannot be repeated. How many PINs are possible? (Use the counting principle.)

26. Weekly pottery classes are offered on Monday, Wednesday, and Thursday. On each of those days there is a class at 5:00 and a class at 7:00. How many classes are offered? (Make a tree diagram.)

Extension

Use after Lesson 13.1

Perform Simulations

GOAL Perform simulations to make predictions.

Key Vocabulary
- **simulation**

A **simulation** is an experiment that you can perform to make predictions about real-world situations.

EXAMPLE 1 Perform a simulation

CONCESSION PRIZES Each time you buy an item from the concession stand at a baseball stadium, you receive a prize coupon, chosen at random. There is an equal chance of winning each prize from the following list: hot dog, popcorn, peanuts, pretzel, ice cream, and small drink. About how many times must you buy an item from the concession stand before you win each prize at least once?

Solution

You can perform a simulation to answer the question.

STEP 1 **Write** each prize on a separate piece of paper. Put the pieces of paper in a container.

STEP 2 **Draw** a piece of paper from the container at random. Record the result in a table like the one shown. Put the piece of paper back in the container. Repeat until you put a tally mark in the last empty cell of the table.

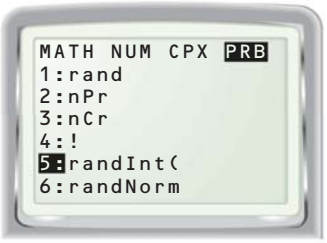
Prize	Hot dog	Popcorn	Peanuts	Pretzel	Ice cream	Small drink
Tally	I	IIII	II	卌	卌 I	II

The sum of all of the tally marks is the number of times you must buy an item from the concession stand before you win each prize at least once.

▶ In this simulation, you must buy an item from the concession stand 20 times.

USING A GRAPHING CALCULATOR You can also use the random integer generator on a graphing calculator to perform simulations.

The random integer generator is found by pressing the MATH key and selecting the PRB menu. It is the fifth item on the list and is displayed as randInt(.

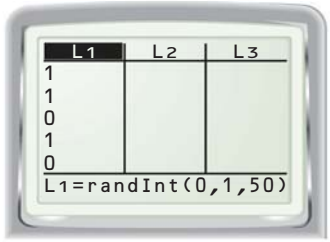


EXAMPLE 2 Perform a simulation using technology

GAME CARDS You receive a game card with every purchase at a sandwich shop. Each card has two circles to scratch. One circle reveals a prize, and the other says "Not a Winner." You cannot claim a prize if you scratch both circles. There is a $\frac{1}{6}$ chance that a card is for a CD, a $\frac{1}{2}$ chance that it is for a drink, and a $\frac{1}{3}$ chance that it is for a sandwich. About how many game cards must you scratch before you win a CD?

Solution

STEP 1 **Use** List 1 to show whether you scratch the circle with the prize. Generate a list of 50 random 1s and 0s. Each 1 means that you scratch the circle with the prize, and each 0 means that you scratch "Not a Winner."

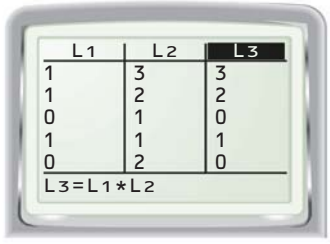


Press STAT and select Edit. Highlight L_1. Enter randInt(0,1,50).

STEP 2 **Use** List 2 to show whether your game card contains the CD as the prize. Generate a list of 50 random integers from 1 to 6. Each 1 represents a prize card with a CD.

Highlight L_2. Enter randInt(1,6,50).

STEP 3 **Compare** the results of your two lists using List 3. Multiply the numbers from List 1 and List 2. Each 0 in List 3 means that you chose the wrong circle, so the prize does not matter. Because $1 \cdot 1 = 1$, you chose the correct circle *and* your card contains the CD prize when you see a 1 in L_3.



Highlight L_3. Enter $L_1 * L_2$.

STEP 4 **Find** the first occurrence of a 1 in List 3. In this simulation, you can see that the first occurrence of a 1 in List 3 happens after 4 trials.

▶ For this simulation, you must scratch 4 game cards before you win a CD.

PRACTICE

EXAMPLE 1 on p. 849 for Exs. 1–3

1. In Example 1, suppose you can receive a prize coupon for nachos in addition to the items listed in the example. About how many times must you buy an item from the concession stand before you win each prize at least once? *Explain* how you found your answer.

EXAMPLE 2 on p. 850 for Exs. 2–3

2. In Example 2, about how many game cards must you scratch before you win one of each prize? *Explain* how you found your answer.
3. In Example 2, there are 3 prizes. *Explain* why the results of the simulation would be inaccurate if you generated random integers from 1 to 3.

13.2 Find Probabilities Using Permutations

Before You used the counting principle.

Now You will use the formula for the number of permutations.

Why? So you can find the number of possible arrangements, as in Ex. 38.

Key Vocabulary
- **permutation**
- ***n* factorial**

A **permutation** is an arrangement of objects in which order is important. For instance, the 6 possible permutations of the letters A, B, and C are shown.

ABC ACB BAC BCA CAB CBA

EXAMPLE 1 Count permutations

Consider the number of permutations of the letters in the word JULY.

a. In how many ways can you arrange all of the letters?

b. In how many ways can you arrange 2 of the letters?

Solution

REVIEW COUNTING PRINCIPLE
For help with using the counting principle, see p. 931.

a. Use the counting principle to find the number of permutations of the letters in the word JULY.

Number of permutations	=	Choices for 1st letter	•	Choices for 2nd letter	•	Choices for 3rd letter	•	Choices for 4th letter
	=	4	•	3	•	2	•	1

$= 24$

▶ There are 24 ways you can arrange all of the letters in the word JULY.

b. When arranging 2 letters of the word JULY, you have 4 choices for the first letter and 3 choices for the second letter.

Number of permutations = Choices for 1st letter • Choices for 2nd letter

$= 4 \cdot 3$

$= 12$

▶ There are 12 ways you can arrange 2 of the letters in the word JULY.

✓ GUIDED PRACTICE for Example 1

1. In how many ways can you arrange the letters in the word MOUSE?

2. In how many ways can you arrange 3 of the letters in the word ORANGE?

FACTORIAL In Example 1, you evaluated the expression $4 \cdot 3 \cdot 2 \cdot 1$. This expression can be written as 4! and is read "4 *factorial*." For any positive integer n, the product of the integers from 1 to n is called ***n* factorial** and is written as $n!$. The value of 0! is defined to be 1.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \text{ and } 0! = 1$$

In Example 1, you also found the permutations of four objects taken two at a time. You can find the number of permutations using the formulas below.

KEY CONCEPT *For Your Notebook*

Permutations

Formulas	Examples
The number of permutations of n objects is given by: $${}_nP_n = n!$$	The number of permutations of 4 objects is: $${}_4P_4 = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$
The number of permutations of n objects taken r at a time, where $r \le n$, is given by: $${}_nP_r = \frac{n!}{(n-r)!}$$	The number of permutations of 4 objects taken 2 at a time is: $${}_4P_2 = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}} = 12$$

EXAMPLE 2 Use a permutations formula

CD RECORDING Your band has written 12 songs and plans to record 9 of them for a CD. In how many ways can you arrange the songs on the CD?

Solution

To find the number of permutations of 9 songs chosen from 12, find ${}_{12}P_9$.

$${}_{12}P_9 = \frac{12!}{(12-9)!}$$ **Permutations formula**

$$= \frac{12!}{3!}$$ **Subtract.**

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}}$$ **Expand factorials. Divide out common factor, 3!.**

$$= 79{,}833{,}600$$ **Multiply.**

▶ There are 79,833,600 ways to arrange 9 songs out of 12.

DIVIDE COMMON FACTORS
When you divide out common factors, remember that 3! is a factor of 12!.

✓ **GUIDED PRACTICE** for Example 2

3. **WHAT IF?** In Example 2, suppose your band has written 15 songs. You will record 9 of them for a CD. In how many ways can you arrange the songs on the CD?

EXAMPLE 3 Find a probability using permutations

PARADE For a town parade, you will ride on a float with your soccer team. There are 12 floats in the parade, and their order is chosen at random. Find the probability that your float is first and the float with the school chorus is second.

Solution

STEP 1 **Write** the number of possible outcomes as the number of permutations of the 12 floats in the parade. This is ${}_{12}P_{12} = 12!$.

STEP 2 **Write** the number of favorable outcomes as the number of permutations of the other floats, given that the soccer team is first and the chorus is second. This is ${}_{10}P_{10} = 10!$.

STEP 3 **Calculate** the probability.

$$P\left(\begin{matrix}\text{soccer team is first}\\ \text{chorus is second}\end{matrix}\right) = \frac{10!}{12!}$$ **Form a ratio of favorable to possible outcomes.**

$$= \frac{\cancel{10!}}{12 \cdot 11 \cdot \cancel{10!}}$$ **Expand factorials. Divide out common factor, 10!.**

$$= \frac{1}{132}$$ **Simplify.**

✓ GUIDED PRACTICE for Example 3

4. **WHAT IF?** In Example 3, suppose there are 14 floats in the parade. Find the probability that the soccer team is first and the chorus is second.

13.2 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 21 and 35
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 30, 33, and 35
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 34

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: An arrangement of objects in which order is important is called a(n) __?__.

2. ★ **WRITING** *Explain* what the notation ${}_9P_2$ means. What is the value of this expression?

EXAMPLES 1 and 2 on pp. 851–852 for Exs. 3–11

COUNTING PERMUTATIONS **Find the number of ways you can arrange (a) all of the letters in the given word and (b) 2 of the letters in the word.**

3. AT
4. TRY
5. GAME
6. CAT
7. WATER
8. ROCK
9. APRIL
10. FAMILY

11. ★ **OPEN-ENDED** *Describe* a real-world situation where the number of possibilities is given by ${}_5P_2$.

EXAMPLE 2 on p. 852 for Exs. 12–30

FACTORIALS AND PERMUTATIONS **Evaluate the expression.**

12. 1! **13.** 3! **14.** 0! **15.** 5!

16. 8! **17.** 10! **18.** 12! **19.** 13!

20. ${}_5P_2$ **21.** ${}_7P_3$ **22.** ${}_9P_1$ **23.** ${}_6P_5$

24. ${}_8P_8$ **25.** ${}_{12}P_0$ **26.** ${}_{30}P_2$ **27.** ${}_{25}P_5$

ERROR ANALYSIS ***Describe*** **and correct the error in evaluating the expression.**

28. ${}_{11}P_7 = \frac{11!}{(11-7)} = \frac{11!}{4} = 9{,}979{,}200$ ✗

29. ${}_5P_3 = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}} = 20$ ✗

30. ★ **MULTIPLE CHOICE** The judges in an art contest award prizes for first, second, and third place out of 11 entries. Which expression gives the number of ways the judges can award first, second, and third place?

Ⓐ $\frac{3!}{11!}$ Ⓑ $\frac{8!}{11!}$ Ⓒ $\frac{11!}{8!}$ Ⓓ $\frac{11!}{3!}$

31. **CHALLENGE** Consider a set of 4 objects and a set of n objects.

a. Are there more permutations of all 4 of the objects or of 3 of the 4 objects? *Justify* your answer using an organized list.

b. In general, are there more permutations of n objects taken n at a time or of n objects taken $n - 1$ at a time? *Justify* your answer using the formula for the number of permutations.

PROBLEM SOLVING

EXAMPLE 2 on p. 852 for Exs. 32–33

32. **MOVIES** Six friends go to a movie theater. In how many different ways can they sit together in a row of 6 empty seats?

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33. ★ **MULTIPLE CHOICE** You plan to visit 4 stores during a shopping trip. In how many orders can you visit these stores?

Ⓐ 4 Ⓑ 16 Ⓒ 24 Ⓓ 256

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EXAMPLE 3 on p. 853 for Exs. 34–38

34. ◆ **MULTIPLE REPRESENTATIONS** You and your friend are two of 4 servers working a shift in a restaurant. The host assigns tables of new diners to the servers in a particular order. This order remains the same, so that all servers are likely to wait on the same number of tables by the end of the shift.

a. **Making a List** List all the possible orders in which the host can assign tables to the servers.

b. **Using a Formula** Use the formula for permutations to find the number of ways in which the host can assign tables to the servers.

c. **Describe in Words** What is the likelihood that you and your friend are assigned the first 2 tables? *Explain* your answer using probability.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

35. ★ **SHORT RESPONSE** Every student in your history class is required to present a project in front of the class. Each day, 4 students make their presentations in an order chosen at random by the teacher. You make your presentation on the first day.

a. What is the probability that you are chosen to be the first or second presenter on the first day? *Explain* how you found your answer.

b. What is the probability that you are chosen to be the second or third presenter on the first day? *Compare* your answer with that in part (a).

36. **HISTORY EXAM** On an exam, you are asked to list 5 historical events in the order in which they occurred. You guess the order of the events at random. What is the probability that you choose the correct order?

37. **SPIRIT** You make 6 posters to hold up at a basketball game. Each poster has a letter of the word TIGERS. You and 5 friends sit next to each other in a row. The posters are distributed at random. What is the probability that TIGERS is spelled correctly when you hold up the posters?

38. **BAND COMPETITION** Seven marching bands will perform at a competition. The order of the performances is determined at random. What is the probability that your school band will perform first, followed by the band from the one other high school in your town?

39. **CHALLENGE** You are one of 10 students performing in a school talent show. The order of the performances is determined at random. The first five performers go on stage before the intermission, while the remaining five performers go on stage after the intermission.

a. What is the probability that you are the last performer before the intermission and your rival performs immediately before you?

b. What is the probability that you are *not* the first performer?

MIXED REVIEW

PREVIEW
Prepare for Lesson 13.3 in Exs. 40–43.

40. You are randomly assigned a day of the week to work an extra shift at your part-time job. Find the probability that you are assigned to work on Saturday. *(p. 843)*

41. You choose a letter at random out of a bag that contains one of each letter of the alphabet. Find the probability that you choose the letter K. *(p. 843)*

42. You roll a number cube. Find the probability that you roll an even number. *(p. 843)*

43. You toss a coin twice. Find the probability that the coin shows tails twice. *(p. 843)*

13.3 Find Probabilities Using Combinations

Before You used permutations to count possibilities.

Now You will use combinations to count possibilities.

Why? So you can find the probability of an event, as in Example 3.

Key Vocabulary
- **combination**

A **combination** is a selection of objects in which order is *not* important. For instance, in a drawing for 3 identical prizes, you would use combinations, because the order of the winners would not matter. If the prizes were different, you would use permutations, because the order would matter.

EXAMPLE 1 Count combinations

Count the combinations of two letters from the list A, B, C, D.

Solution

List all of the permutations of two letters in the list A, B, C, D. Because order is not important in a combination, cross out any duplicate pairs.

AB	AC	AD	~~BA~~	BC	BD
~~CA~~	~~CB~~	CD	~~DA~~	~~DB~~	~~DC~~

BD and DB are the same pair.

▶ There are 6 possible combinations of 2 letters from the list A, B, C, D.

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✓ GUIDED PRACTICE for Example 1

1. Count the combinations of 3 letters from the list A, B, C, D, E.

COMBINATIONS In Example 1, you found the number of combinations of objects by making an organized list. You can also find the number of combinations using the following formula.

KEY CONCEPT *For Your Notebook*

Combinations

Formula

The number of combinations of n objects taken r at a time, where $r \le n$, is given by:

$$_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

Example

The number of combinations of 4 objects taken 2 at a time is:

$$_4C_2 = \frac{4!}{(4-2)! \cdot 2!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!} \cdot (2 \cdot 1)} = 6$$

EXAMPLE 2 Use the combinations formula

LUNCH MENU You order a sandwich at a restaurant. You can choose 2 side dishes from a list of 8. How many combinations of side dishes are possible?

Solution

The order in which you choose the side dishes is not important. So, to find the number of combinations of 8 side dishes taken 2 at a time, find ${}_8C_2$.

$${}_8C_2 = \frac{8!}{(8-2)! \cdot 2!}$$ **Combinations formula**

$$= \frac{8!}{6! \cdot 2!}$$ **Subtract.**

$$= \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot (2 \cdot 1)}$$ **Expand factorials. Divide out common factor, 6!.**

$$= 28$$ **Simplify.**

▶ There are 28 different combinations of side dishes you can order.

EXAMPLE 3 Find a probablity using combinations

PHOTOGRAPHY A yearbook editor has selected 14 photos, including one of you and one of your friend, to use in a collage for the yearbook. The photos are placed at random. There is room for 2 photos at the top of the page. What is the probability that your photo and your friend's photo are the two placed at the top of the page?

Solution

STEP 1 **Write** the number of possible outcomes as the number of combinations of 14 photos taken 2 at a time, or ${}_{14}C_2$, because the order in which the photos are chosen is not important.

$${}_{14}C_2 = \frac{14!}{(14-2)! \cdot 2!} = \frac{14!}{12! \cdot 2!} = \frac{14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!} \cdot (2 \cdot 1)} = 91$$

STEP 2 **Find** the number of favorable outcomes. Only one of the possible combinations includes your photo and your friend's photo.

STEP 3 **Calculate** the probability.

P(your photo and your friend's photos are chosen) $= \frac{1}{91}$

✓ GUIDED PRACTICE for Examples 2 and 3

2. **WHAT IF?** In Example 2, suppose you can choose 3 side dishes out of the list of 8 side dishes. How many combinations are possible?

3. **WHAT IF?** In Example 3, suppose there are 20 photos in the collage. Find the probability that your photo and your friend's photo are the two placed at the top of the page.

13.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7 and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14–20, and 25

SKILL PRACTICE

1. VOCABULARY Copy and complete: A(n) __?__ is a selection of objects in which order is not important.

2. ★ WRITING *Explain* how a combination differs from a permutation.

EXAMPLE 1 on p. 856 for Exs. 3, 4

3. COMBINATIONS How many combinations of 3 letters from the list A, B, C, D, E, F are possible?

4. ERROR ANALYSIS *Describe* and correct the error in listing all of the possible combinations of 2 letters from the list A, B, C.

AB	BA	CA
AC	BC	CB

EXAMPLE 2 on p. 857 for Exs. 5–15

5. ERROR ANALYSIS *Describe* and correct the error in evaluating ${}_9C_4$.

$${}_9C_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 3024$$

COMBINATIONS Evaluate the expression.

6. ${}_5C_1$ **7.** ${}_8C_5$ **8.** ${}_9C_9$ **9.** ${}_8C_6$

10. ${}_{12}C_3$ **11.** ${}_{11}C_4$ **12.** ${}_{15}C_8$ **13.** ${}_{20}C_5$

14. ★ MULTIPLE CHOICE What is the value of ${}_{10}C_6$?

Ⓐ 7 Ⓑ 60 Ⓒ 210 Ⓓ 151,200

15. ★ MULTIPLE CHOICE You have the first season of your favorite television show on a set of DVDs. The set contains 13 episodes. You have time to watch 3 episodes. How many combinations of 3 episodes can you watch?

Ⓐ 286 Ⓑ 572 Ⓒ 1716 Ⓓ 589,680

★ SHORT RESPONSE In Exercises 16–19, tell whether the question can be answered using *combinations* or *permutations*. *Explain* your choice, then answer the question.

16. Four students from your class of 120 students will be selected to organize a fundraiser. How many groups of 4 students are possible?

17. Ten students are auditioning for 3 different roles in a play. In how many ways can the 3 roles be filled?

18. To complete an exam, you must answer 8 questions from a list of 10 questions. In how many ways can you complete the exam?

19. In how many ways can 5 people sit in a car that holds 5 passengers?

20. ★ WRITING Which is greater, ${}_6P_r$ or ${}_6C_r$? *Justify* your answer.

21. REASONING Write an equation that relates ${}_nP_r$ and ${}_nC_r$. *Explain* your reasoning.

22. CHALLENGE Prove that ${}_nC_r = {}_nC_{n-r}$. *Explain* why this makes sense.

PROBLEM SOLVING

EXAMPLE 2 on p. 857 for Ex. 23

23. RESTAURANT You are ordering a burrito with 2 main ingredients and 3 toppings. The menu below shows the possible choices. How many different burritos are possible?

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EXAMPLE 3 on p. 857 for Exs. 24–26

24. WORK SCHEDULE You work 3 evenings each week at a bookstore. Your supervisor assigns you 3 evenings at random from the 7 possibilities. What is the probability that your schedule this week includes working on Friday?

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25. ★ **SHORT RESPONSE** On a television game show, 9 members of the studio audience are randomly selected to be eligible contestants.

a. Six of the 9 eligible contestants are randomly chosen to play a game on the stage. How many combinations of 6 players from the group of eligible contestants are possible?

b. You and your two friends are part of the group of 9 eligible contestants. What is the probability that all three of you are chosen to play the game on stage? *Explain* how you found your answer.

26. REPRESENTATIVES Your teacher chooses 2 students at random to represent your homeroom. The homeroom has a total of 30 students, including your best friend. What is the probability that you and your best friend are chosen? What is the probability that you are chosen first and your best friend is chosen second? Which event is more likely to occur?

27. CHALLENGE There are 30 students in your class. Your science teacher will choose 5 students at random to complete a group project. Find the probability that you and your 2 best friends in the science class are chosen to work in the group. *Explain* how you found your answer.

MIXED REVIEW

PREVIEW Prepare for Lesson 13.4 in Exs. 28–32.

Find the product. *(p. 915)*

28. $\frac{1}{6} \cdot \frac{4}{5}$ **29.** $\frac{4}{25} \cdot \frac{7}{24}$ **30.** $\frac{13}{30} \cdot \frac{5}{26}$ **31.** $\frac{7}{24} \cdot \frac{4}{51}$

32. You roll a number cube. What is the probability that you roll a multiple of 3? *(p. 843)*

33. In how many ways can you arrange 3 letters from the list P, Q, R, S, T, L? *Explain* how you found your answer. *(p. 851)*

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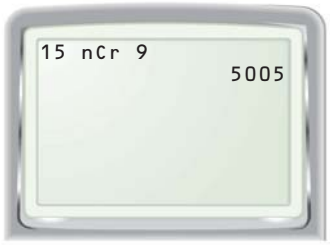
13.3 Find Permutations and Combinations

QUESTION How can you find combinations and permutations using a graphing calculator?

EXAMPLE 1 Find the number of combinations

STARTERS There are 15 players on your softball team, but only 9 of them can be the starting players in one game. How many combinations of starting players are possible?



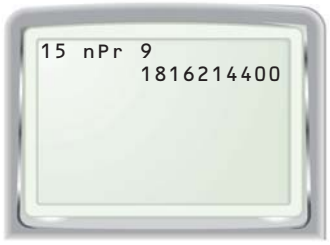
Solution

You are finding ${}_nC_r$ where $n = 15$ and $r = 9$. Enter 15 for n.
Press MATH. Go to the PRB menu and select ${}_nC_r$.
Then enter 9 for r.

▶ There are 5005 possible combinations of starting players.

EXAMPLE 2 Find the number of permutations

BATTING ORDER Before each softball game, your coach announces the batting order of the 9 starting players. This is the order in which the starting players will bat. How many batting orders can be formed using 9 players on your team of 15 players?



Solution

You are finding ${}_nP_r$ where $n = 15$ and $r = 9$. Enter 15 for n.
Press MATH. Go to the PRB menu and select ${}_nP_r$.
Then enter 9 for r.

▶ There are 1,816,214,400 possible batting orders.

PRACTICE

Evaluate the expression.

1. ${}_7C_4$
2. ${}_6C_6$
3. ${}_{10}C_3$
4. ${}_{16}C_8$
5. ${}_9P_5$
6. ${}_7P_6$
7. ${}_{11}P_8$
8. ${}_{12}P_5$

9. **GROUP PROJECT** Your teacher selects 3 students from a class of 28 students to work on a project in a group. Within the group, one member must be the writer, one must be the researcher, and one must be the presenter.
   a. How many different groups of 3 can your teacher select?
   b. After the group is formed, in how many ways can the roles in the group be assigned?

13.4 Find Probabilities of Compound Events

Before You found the probability of a simple event.

Now You will find the probability of a compound event.

Why? So you can analyze scientific data, as in Ex. 23.

Key Vocabulary
- **compound event**
- **mutually exclusive events**
- **overlapping events**
- **independent events**
- **dependent events**

REVIEW VENN DIAGRAMS
For help with using Venn diagrams, see p. 930.

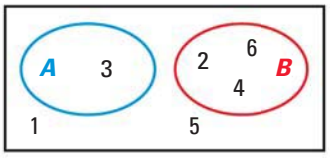
A **compound event** combines two or more events, using the word *and* or the word *or*. To find the probability that either event A or event B occurs, determine how the events are related. **Mutually exclusive events** have no common outcomes. **Overlapping events** have at least one common outcome.

For instance, suppose you roll a number cube.

Mutually Exclusive Events

Event *A*: Roll a 3.

Event *B*: Roll an even number.



Set A has **1** number, and set B has **3** numbers.

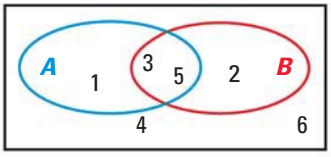
$P(3 \text{ or even}) = \frac{1}{6} + \frac{3}{6}$

$\boldsymbol{P(A \text{ or } B) = P(A) + P(B)}$

Overlapping Events

Event *A*: Roll an odd number.

Event *B*: Roll a prime number.



Set A has **3** numbers, and set B has **3** numbers. There are **2** numbers in both sets.

$P(\text{odd or prime}) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6}$

$\boldsymbol{P(A \text{ or B}) = P(A) + P(B) - P(A \text{ and } B)}$

EXAMPLE 1 Find the probability of *A* or *B*

You roll a number cube. Find the probability that you roll a 2 or an odd number.

Solution

Because 2 is an even number, rolling a 2 and rolling an odd number are mutually exclusive events.

$$P(2 \text{ or odd}) = P(2) + P(\text{odd})$$
$$= \frac{1}{6} + \frac{3}{6}$$
$$= \frac{4}{6}$$
$$= \frac{2}{3}$$

EXAMPLE 2 Find the probability of *A* or *B*

You roll a number cube. Find the probability that you roll an even number or a prime number.

Solution

Because 2 is both an even number and a prime number, rolling an even number and rolling a prime number are overlapping events. There are 3 even numbers, 3 prime numbers, and 1 number that is both.

$$P(\text{even or prime}) = P(\text{even}) + P(\text{prime}) - P(\text{even and prime})$$

$$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6}$$

$$= \frac{5}{6}$$

✓ GUIDED PRACTICE for Examples 1 and 2

1. You roll a number cube. Find the probability that you roll a 2 or a 5.
2. You roll a number cube. Find the probability that you roll a number less than 4 or an odd number.

INDEPENDENT AND DEPENDENT EVENTS To find the probability that event *A* and event *B* both occur, determine how the events are related. Two events are **independent events** if the occurrence of one event has no effect on the occurrence of the other. Two events are **dependent events** if the occurrence of one event affects the occurrence of the other.

For instance, consider the probability of choosing a green marble and then a blue marble from the bag shown. If you choose one marble and replace it before choosing the second, then the events are independent. If you do not replace the first marble, then the sample space has changed, and the events are dependent.

Independent Events

With replacement:

$$P(\text{green and blue}) = \frac{4}{7} \cdot \frac{1}{7} = \frac{4}{49}$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Dependent Events

Without replacement:

$$P(\text{green and blue}) = \frac{4}{7} \cdot \frac{1}{6} = \frac{2}{21}$$

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$$

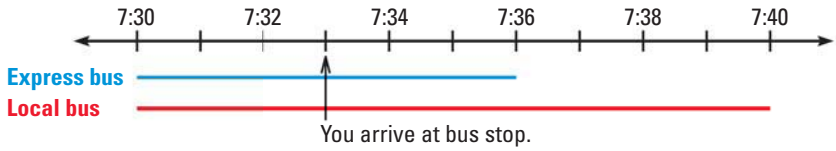
EXAMPLE 3 Find the probability of *A* and *B*

BUS SCHEDULE You take a city bus from your neighborhood to a location within walking distance of your school. The express bus arrives at your neighborhood between 7:30 and 7:36. The local bus arrives at your neighborhood between 7:30 and 7:40. You arrive at the bus stop at 7:33. Find the probability that you have missed both the express bus and the local bus.

ANOTHER WAY
For alternative methods for solving the problem in Example 3, turn to page 868 for the **Problem Solving Workshop**.

Solution

The events are independent. The arrival of one bus does not affect the arrival of the other bus.



There are 6 minutes when the express bus can arrive. You are not at the bus stop for 3 of those minutes.

$P(\text{you miss express bus}) = \frac{3}{6} = \frac{1}{2}$

There are 10 minutes when the local bus can arrive. You are not at the bus stop for 3 of those minutes.

$P(\text{you miss local bus}) = \frac{3}{10}$

Multiply the probabilities of the two events:

$$P(\text{you miss both buses}) = \frac{1}{2} \cdot \frac{3}{10} = \frac{3}{20}$$

▶ The probability that you miss the express bus and the local bus is $\frac{3}{20}$.

EXAMPLE 4 Find the probability of *A* and *B*

PEN COLORS A box contains 3 blue pens and 5 black pens. You choose one pen at random, do not replace it, then choose a second pen at random. What is the probability that both pens are blue?

Solution

Because you do not replace the first pen, the events are dependent. Before you choose a pen, there are 8 pens, and 3 of them are blue. After you choose a blue pen, there are 7 pens left and 2 of them are blue.

$$P(\text{blue and then blue}) = P(\text{blue}) \cdot P(\text{blue given blue})$$

$$= \frac{3}{8} \cdot \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$$

✓ GUIDED PRACTICE for Examples 3 and 4

3. **MARBLES** A bag contains 4 red, 5 green, and 2 blue marbles. You randomly draw 2 marbles, one at a time. Find the probabililty that both are red if:

   a. you replace the first marble.

   b. you do not replace the first marble.

13.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5 and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 8, 13, 20, and 24

◆ = **MULTIPLE REPRESENTATIONS** Ex. 25

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The probability of __?__ events is found using the formula $P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$.

2. ★ **WRITING** *Explain* how overlapping events differ from mutually exclusive events.

EXAMPLES 1 and 2 on pp. 861–862 for Exs. 3–8

PROBABILITY OF A OR B **In Exercises 3–6, you roll a number cube. Tell whether the events *A* and *B* are *mutually exclusive* or *overlapping*. Then find *P*(*A* or *B*).**

3. **Event *A*:** Roll a 6.
   Event *B*: Roll a prime number.

4. **Event *A*:** Roll an even number.
   Event *B*: Roll a 5.

5. **Event *A*:** Roll an odd number.
   Event *B*: Roll a number less than 5.

6. **Event *A*:** Roll a multiple of 3.
   Event *B*: Roll an even number.

7. **ERROR ANALYSIS** A bag contains 7 yellow marbles, 4 red marbles, and 5 blue marbles. *Describe* and correct the error in finding the probability that you randomly draw a yellow or blue marble.

$$P(\text{yellow or blue}) = P(\text{yellow}) \cdot P(\text{blue}) = \frac{7}{16} \cdot \frac{5}{16} = \frac{35}{256}$$

8. ★ **MULTIPLE CHOICE** A bag contains tiles with the numbers 1–10 on them. You randomly choose a tile from the bag. What is the probability that you choose an even number or a number less than 5?

   Ⓐ 0.7 Ⓑ 0.8 Ⓒ 0.9 Ⓓ 1

EXAMPLES 3 and 4 on p. 863 for Exs. 9–12

PROBABILITY OF A AND B **In Exercises 9–12, tell whether the events *A* and *B* are *dependent* or *independent*. Then find *P*(*A* and *B*).**

9. You roll two number cubes.
   Event *A*: You roll a 2 first.
   Event *B*: You roll a 5 second.

10. You write each of the letters of the word BIOLOGY on pieces of paper and place them in a bag. You randomly draw one letter, do not replace it, then randomly draw a second letter.
    Event *A*: The first letter is O.
    Event *B*: The second letter is B.

11. You flip a coin and roll a number cube.
    Event *A*: The coin shows heads.
    Event *B*: The number cube shows 2.

12. A box contains 3 milk chocolates, 3 white chocolates, and 4 dark chocolates. You choose a chocolate at random, eat it, then choose a second chocolate at random.
    Event *A*: You choose a dark chocolate.
    Event *B*: You choose a dark chocolate.

13. ★ **MULTIPLE CHOICE** A vase holds 7 red roses and 5 pink roses. You randomly choose a rose, place it in a different vase, then randomly choose another rose. What is the approximate probability that both the first and second roses are red?

Ⓐ 0.29 Ⓑ 0.32 Ⓒ 0.34 Ⓓ 0.37

CHESS PIECES In Exercises 14–17, consider a bag that contains all of the chess pieces in a set, as shown in the diagram.

	King	Queen	Bishop	Rook	Knight	Pawn
Black	1	1	2	2	2	8
White	1	1	2	2	2	8

14. You choose one piece at random. Find the probability that you choose a black piece or a queen.

15. You choose one piece at random, replace it, then choose a second piece at random. Find the probability that you choose a rook, then a bishop.

16. You choose one piece at random, do not replace it, then choose a second piece at random. Find the probability that you choose a king, then a pawn.

17. **ERROR ANALYSIS** *Describe* and correct the error in finding the probability that you randomly choose a pawn and a second pawn, without replacement.

$$P(\text{pawn and pawn}) = P(\text{pawn}) \cdot P(\text{pawn})$$
$$= \frac{16}{32} \cdot \frac{16}{32} = \frac{1}{4}$$

In Exercises 18 and 19, use the following information. Two mutually exclusive events for which one or the other must occur are called *complementary* events. If events A and B are complementary events, then $P(A) + P(B) = 1$.

18. **WEATHER** A local meteorologist reports that there is a 70% chance of rain tomorrow. What is the probability that it will *not* rain tomorrow?

19. **BASKETBALL** You make 31% of your attempted 3-point shots. What is the probability that you miss your next attempted 3-point shot?

20. ★ **WRITING** You write the letters of the word WISDOM on pieces of paper and place them in a bag. You randomly choose 2 letters from the bag at the same time. *Explain* whether these events are independent or dependent. What is the probability that you choose the letters S and D?

21. **CHALLENGE** The sections of the spinner shown all have the same area. You spin the spinner.

a. Find the probability that the spinner stops on red *or* a prime number *or* a multiple of 3. You may want to draw a Venn diagram to find the answer.

b. Write a general formula for $P(A \text{ or } B \text{ or } C)$ where A, B, and C are overlapping events. *Explain* your reasoning.

PROBLEM SOLVING

EXAMPLES 3 and 4 on p. 863 for Exs. 22–23

22. CONTEST You can win concert tickets from a radio station if you are the first person to call when the song of the day is played, or if you are the first person to correctly answer the trivia question. The song of the day is played between 5:00 and 5:30 P.M. The trivia question is asked between 5:15 and 5:45 P.M. You begin listening to the radio station at 5:20. Find the probability that you miss the song of the day and the trivia question.

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23. WALRUS When a walrus forages for food, it waves its flipper to move sediment 70% of the time. When using the flipper wave technique, a walrus uses its right flipper 89% of the time. Find the probability that a walrus foraging for food uses a flipper and it is the right flipper.

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EXAMPLES 1, 2, 3, and 4 on pp. 861–863 for Ex. 24

24. ★ SHORT RESPONSE A survey of 887,403 households found that 270,658 households have a dog, 326,591 have a cat, and 81,641 have both.

a. What is the probability that one of the households surveyed, chosen at random, has a dog and a cat?

b. What is the probability that one of the households surveyed, chosen at random, has a dog or a cat?

c. *Explain* how your answers to parts (a) and (b) are related.

EXAMPLES 1 and 2 on pp. 861–862 for Ex. 25

25. MULTIPLE REPRESENTATIONS You have student government meetings on Monday and Wednesday. You tutor in the morning on Monday, Thursday, Friday, and Saturday.

a. Making a Table Make a table that shows your schedule for the week.

b. Drawing a Diagram Make a Venn diagram that shows the days of the week that you participate in each activity.

c. Using a Formula Your class is taking a field trip that could be scheduled for any day of the week. Find the probability that it is scheduled for a day when you tutor or have a student government meeting.

26. EARTH SCIENCE The table shows the ranges of annual mean temperature and precipitation for 57 cities in the U.S. Find the probability that a city in this study has an annual mean temperature in the range 39°F–52°F or an annual precipitation in the range 0–24 inches.

Precipitation (inches)	Temperature (degrees Fahrenheit)	
	39–52	53–66
0–24	7	7
25–49	21	22

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

27. **CHALLENGE** You have 5 tickets to a play. You invite 4 friends to see the play. You hand out the tickets at random. One ticket is for an aisle seat, and the other tickets are for the next 4 seats in the row.

   a. What is the probability that you will get the aisle seat?

   b. What is the probability that you will get the aisle seat and your best friend will get the ticket for the seat next to you?

   c. *Explain* how you could solve the problem in part (b) using permutations.

MIXED REVIEW

PREVIEW
Prepare for Lesson 13.5 in Exs. 28–29.

In Exercises 28 and 29, the spinner shown has sections with equal area. *(p. 843)*

28. You flip a coin and spin the spinner. How many possible outcomes are in the sample space? List the possible outcomes.

29. You roll a number cube and spin the spinner. How many possible outcomes are in the sample space? List the possible outcomes.

Evaluate the expression.

30. ${}_5P_3$ *(p. 851)*

31. ${}_{15}P_0$ *(p. 851)*

32. ${}_{15}C_0$ *(p. 856)*

33. ${}_5C_3$ *(p. 856)*

34. **ELECTIVES** Your school offers 10 elective courses each semester. You have time in your schedule for 2 of these courses. How many combinations of 2 elective courses can you choose? *(p. 856)*

QUIZ *for Lessons 13.1–13.4*

1. **MARBLES** A bag contains 16 red marbles and 8 white marbles. You select a marble at random. *(p. 843)*

   a. What is the probability that you select a red marble?

   b. What are the odds in favor of selecting a red marble?

2. **PASSWORD** The password for an e-mail account is the word FISH followed by a 3-digit number. The 3-digit number contains the digits 1, 2, and 3. How many different passwords are possible? *(p. 851)*

3. **SHUFFLE** A CD plays on random shuffle. The CD has 12 songs on it. Your CD player selects a song at random, plays it, then selects a second song at random. No song is repeated until every song has been played. What is the probability that song 3 is played first and song 1 is played second? *(p. 851)*

Evaluate the expression.

4. ${}_5P_4$ *(p. 851)*

5. ${}_8P_5$ *(p. 851)*

6. ${}_5C_2$ *(p. 856)*

7. ${}_8C_5$ *(p. 856)*

8. **NUMBER TILES** Tiles numbered 1–30 are placed in a bag. You select a tile at random. Find the probability that you select an odd number or a prime number. Are the events mutually exclusive or overlapping? *Explain*. *(p. 861)*

PROBLEM SOLVING WORKSHOP
LESSON 13.4

Using ALTERNATIVE METHODS

Another Way to Solve Example 3, page 863

MULTIPLE REPRESENTATIONS In Example 3 on page 863, you saw how to solve the problem about a bus schedule by using a number line and a formula. You can also solve the problem by performing a simulation or using geometry.

PROBLEM

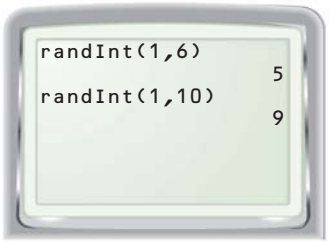
BUS SCHEDULE You take a city bus from your neighborhood to a location within walking distance of your school. The express bus arrives at your neighborhood between 7:30 and 7:36. The local bus arrives at your neighborhood between 7:30 and 7:40. You arrive at the bus stop at 7:33. Find the probability that you have missed both the express bus and the local bus.

METHOD 1

Performing a Simulation One alternative approach is to perform a simulation.

STEP 1 **Read** the problem. Notice that there is a 6 minute interval when the express bus could arrive and a 10 minute interval when the local bus could arrive. Let 1 represent the first minute, from 7:30 to 7:31, that a bus could arrive. Let 2 represent the second minute, from 7:31 to 7:32, that a bus could arrive. Continue to number the minutes when a bus could arrive.

STEP 2 **Generate** random integers. Use a graphing calculator to generate a random integer from 1 to 6. This number represents the minute that the express bus arrives. Then generate a random integer from 1 to 10. This number represents the minute that the local bus arrives. Perform this simulation 10 times.



You are not at the bus stop until the fourth minute, so if both numbers that you generate are less than 4, then you miss both buses.

First number	5	4	2	5	2	1	2	3	3	1
Second number	9	1	1	8	4	7	9	10	6	2
Miss both buses?	No	No	Yes	No	No	No	No	No	No	Yes

STEP 3 **Find** the experimental probability that you miss both buses.

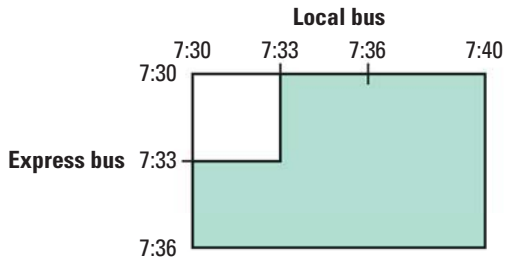
$$P(\text{miss both buses}) = \frac{2}{10} = \frac{1}{5}$$

METHOD 2

Using Geometry Another approach is to use geometry. Use the formula for the area of a rectangle to find the number of possible outcomes and the number of favorable outcomes.

STEP 1 **Draw** a rectangle whose side lengths represent the number of minutes that each bus could arrive.

STEP 2 **Draw** a square within the rectangle to represent the number of minutes that you are *not* at the bus stop.



STEP 3 **Calculate** the area of the rectangle that represents the time a bus could arrive. Also calculate the area of the square that represents the time that you are *not* at the bus stop.

Time a bus could arrive:
$A = 6 \cdot 10 = 60$

Time you are *not* at bus stop:
$A = 3 \cdot 3 = 9$

STEP 4 **Find** the probability that you miss both buses by forming the ratio of the areas from step 2.

$$P(\text{miss both buses}) = \frac{9}{60} = \frac{3}{20}$$

PRACTICE

1. **WHAT IF?** In the problem on page 868, suppose you arrive at 7:34. What is the probability that you miss both buses?

2. **VISITING FRIENDS** Two friends are planning to visit you this evening. You expect one friend to arrive at your house between 7:00 and 7:30 P.M. You expect the other friend to arrive between 7:10 and 7:20 P.M. You have to run an errand from 7:00 until 7:15 P.M. What is the probability that you are home when both friends arrive? Solve this problem using two different methods.

3. **WHAT IF?** In Exercise 2, suppose a third friend plans to visit you this evening. This friend plans to arrive at your house between 7:00 and 7:20 P.M. What is the probability that you are home when all three of your friends arrive? *Explain* how you found your answer.

4. **RAFFLE** You enter two different raffles during your neighborhood's street fair. The winner of the first raffle will be announced between 6:00 and 6:30 P.M. The winner of the second raffle will be announced between 6:15 and 6:45 P.M. You leave the fair at 5:00 P.M. and return at 6:20 P.M. What is the probability that you hear the winner of each raffle announced? Solve this problem using two different methods.

5. **ERROR ANALYSIS** A student solved the problem in Exercise 4 as shown. *Describe* and correct the error.

$$P(\text{hear both winners}) = \frac{\text{Favorable time}}{\text{Total time}} = \frac{10 \text{ minutes}}{30 \text{ minutes}} = \frac{1}{3}$$

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 13.1–13.4

1. **MULTI-STEP PROBLEM** There are 5743 known amphibian species in the world. Of these, 1856 species are judged to be at risk of extinction, and another 113 species may already be extinct.

   a. Find the probability that an amphibian species chosen at random is at risk of extinction or may already be extinct.

   b. Find the probability that two different amphibian species, each chosen at random, are at risk of extinction.

The Puerto Rican crested toad is at risk of extinction.

2. **MULTI-STEP PROBLEM** You are ordering an omelet with two ingredients. You can choose from the following list: cheese, mushrooms, onions, tomatoes, peppers, sausage, ham, and steak.

   a. Make an organized list of all the possible omelets that you can order.

   b. Use a permutation or combination formula to find the number of possible omelets.

3. **MULTI-STEP PROBLEM** In NCAA women's basketball tournaments from 1982 to 2003, teams seeded, or ranked, number one have won 283 games and lost 71 games in the tournament. Suppose a team is chosen at random from all those that have been seeded number one.

   a. What is the probability that the team won a game in the tournament?

   b. What are the odds in favor of the team's having won a game in the tournament?

4. **SHORT RESPONSE** A meteorologist reports that there is a 15% chance of snow tomorrow. What are the odds in favor of snow tomorrow? *Explain* how you found your answer.

5. **OPEN-ENDED** *Describe* a real-world situation in which the number of possible arrangements is given by ${}_{10}P_2$.

6. **SHORT RESPONSE** In the United States there are 21 states (not including Washington, D.C.) with teams in the National Football League and 17 states with Major League Baseball teams. There are 15 states that have both types of teams. Suppose a state is chosen at random.

   a. Find the probability that the state has either a National Football League team or a Major League Baseball team.

   b. There are 21 states that have a team in the National Basketball Association. What additional information would you need in order to find the probability that the state chosen at random has either a team in the National Basketball Association or a Major League Baseball team? *Explain* your reasoning.

7. **EXTENDED RESPONSE** A survey asked a total of 400 students, 100 male students and 100 female students who were 13 and 15 years old, about their eating habits. The table shows the numbers of students who said that they eat fruit every day.

	13 years old	15 years old
Male	60	53
Female	61	58

   a. Find the probability that a female student, chosen at random from the students surveyed, eats fruit every day.

   b. Find the probability that a 15-year-old student, chosen at random from the students surveyed, eats fruit every day.

   c. You select a student at random from the students surveyed. Find the odds against the student's eating fruit every day. *Explain* your reasoning.

8. **GRIDDED ANSWER** A music club gives you 6 free CDs for joining. You would like to own 11 of the free CDs that are offered. How many combinations of 6 CDs from the 11 CDs can you choose?

13.5 Analyze Surveys and Samples

Before You found experimental probabilities.

Now You will identify populations and sampling methods.

Why? So you can analyze surveys of sports fans, as in Ex. 15.

Key Vocabulary
- **survey**
- **population**
- **sample**
- **biased sample**
- **biased question**

A **survey** is a study of one or more characteristics of a group. The entire group you want information about is called a **population**. You may find it difficult to survey an entire population. Instead, you can survey a **sample**, which is a part of the population. Five types of samples are listed below.

KEY CONCEPT *For Your Notebook*

Sampling Methods

In a **random sample**, every member of the population has an equal chance of being selected.

In a **stratified random sample**, the population is divided into distinct groups. Members are selected at random from each group.

In a **systematic sample**, a rule is used to select members of the population.

In a **convenience sample**, only members of the population who are easily accessible are selected.

In a **self-selected sample**, members of the population select themselves by volunteering.

EXAMPLE 1 Classify a sampling method

EMPLOYEE SAFETY The owners of a company with several factories conduct a survey to determine whether employees are informed about safety regulations. At each factory, 50 employees are chosen at random to complete the survey. Identify the population and classify the sampling method.

Solution

The population is all company employees. Because the population is divided into distinct groups (individual factories), with employees chosen at random from each group, the sample is a stratified random sample.

✓ GUIDED PRACTICE for Example 1

1. **WHAT IF?** In Example 1, suppose the owners survey each employee whose last name begins with M. Classify the sampling method.

BIASED SAMPLES A sample chosen for a survey should be representative of the population. A **biased sample** is a sample that is not representative. In a biased sample, parts of the population may be over-represented or under-represented.

Random samples and stratified random samples (as in Example 1) are the most likely types of samples to be representative. A systematic sample may be representative if the rule used to choose individuals is not biased.

EXAMPLE 2 Identify a potentially biased sample

In Example 1, suppose the owners question 50 workers chosen at random from one factory. Is the method likely to result in a biased sample?

Solution

Workers at other factories may hold significantly different opinions, so the method may result in a biased sample.

BIASED QUESTIONS A question that encourages a particular response is a **biased question**. Survey questions should be worded to avoid bias.

EXAMPLE 3 Identify potentially biased questions

Tell whether the question is potentially biased. Explain your answer. If the question is potentially biased, rewrite it so that it is not.

a. Don't you agree that the voting age should be lowered to 16 because many 16-year-olds are responsible and informed?

b. Do you think the city should risk an increase in pollution by allowing expansion of the Northern Industrial Park?

Solution

a. This question is biased because it suggests that lowering the voting age is a good thing to do. An unbiased question is "Do you think the voting age should be lowered to 16?"

b. This question is biased because it suggests that the proposed expansion will be bad for the environment. An unbiased question is "Do you think the city should allow expansion of the Northern Industrial Park?"

✓ GUIDED PRACTICE for Examples 2 and 3

2. SOCCER In a survey about Americans' interest in soccer, the first 25 people admitted to a high school soccer game were asked, "How interested are you in the world's most popular sport, soccer?"

a. Is the sampling method likely to result in a biased sample? *Explain*.

b. Is the question potentially biased? *Explain* your answer. If the question is potentially biased, rewrite it so that it is not.

13.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3 and 15

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, and 17

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In a(n) __?__ sample, participants are chosen using a rule.

2. ★ **WRITING** *Explain* how a sample is related to a population.

POPULATIONS AND SAMPLES **In Exercises 3–5, identify the population and classify the sampling method.**

EXAMPLE 1 on p. 871 for Exs. 3–6

3. **RESTAURANT SERVICE** A restaurant manager wants to evaluate the restaurant's quality of service. Diners are given mail-in comment cards.

4. **EXTRACURRICULAR ACTIVITIES** Your school wants to know if students are satisfied with the school's extracurricular activities. In each grade, every tenth student on an alphabetized list is surveyed.

5. **CUSTOMER SATISFACTION** An airline wants to gather information on passenger satisfaction during a flight. A computer randomly selects 30 passengers to complete a survey.

6. ★ **MULTIPLE CHOICE** Scientists wanted to gather information about the birds in a particular region. They chose observation sites and asked bird watchers at those sites to record the number and types of birds they saw in 3 minutes. What population was being studied?

   (A) Birds (B) Sites (C) Scientists (D) Bird watchers

EXAMPLE 2 on p. 872 for Exs. 7–8

BIASED SAMPLES **Tell whether the sampling method used is likely to result in a biased sample. *Explain*.**

7. **NEIGHBORHOOD WATCH** A family wants to gather information from other residents on their street about forming a neighborhood watch. They survey every third house on both sides of the street.

8. **NURSE SURVEY** The American Nurses Association wanted to gather information about the working environment for nurses in hospitals. A survey for nurses was posted on the association's website.

EXAMPLE 3 on p. 872 for Exs. 9–11

BIASED QUESTIONS **In Exercises 9 and 10, tell whether the question is potentially biased. *Explain* your answer.**

9. Do you support the incumbent's tax plan or the challenger's tax plan?

10. Do you prefer the ease of shopping online or the fun of going to a mall?

11. **ERROR ANALYSIS** *Describe* and correct the error in revising the survey question "Don't you think the minimum driving age should be lower?" so that it is not biased.

> Not biased:
> Is the minimum driving age too high or too low?

12. **CHALLENGE** Two toothpaste manufacturers each claim that 4 out of every 5 dentists use their brand exclusively. Both manufacturers can support their claims with survey results. *Explain* how this is possible.

PROBLEM SOLVING

EXAMPLES 2 and 3 on p. 872 for Exs. 13–16

In Exercises 13 and 14, explain why the question is biased. Then rewrite it so that it is not.

13. Don't you agree that the school needs a new athletic field more than a new science lab?

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14. Would you pay even higher concert ticket prices to finance a new arena?

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15. **BASEBALL** Every Major League Baseball (MLB) season, players are chosen to represent the two leagues in an All-Star game. At each MLB park, fans are given ballots to vote for their favorite players. Are the ballots collected at the Chicago Cubs' park, Wrigley Field, necessarily representative of the opinions of all Chicago Cubs fans? *Explain*.

16. **WATER SAMPLING** Scientists designed a project in which students performed tests on local water sources each day. Students from 18 countries participated in the project. The results of the survey were used to assess the quality of the world's fresh water. Is the sample likely to be biased? *Explain*.

17. ★ **SHORT RESPONSE** You plan to report on the academic performance of students in your school for your school newspaper.

a. *Describe* how you could choose a representative sample.

b. Write an unbiased question you could use to collect information on how many hours per night a student studies. *Explain* why your question is unbiased.

18. **CHALLENGE** A systematic sample of a population is used for a survey containing unbiased questions. *Explain* how it is possible for the survey to be biased. *Describe* a situation in which this might occur.

MIXED REVIEW

PREVIEW Prepare for Lesson 13.6 in Exs. 19–23.

Order the numbers from least to greatest. *(p. 909)*

19. 0.02, 0.015, 0.021, 0.012 **20.** 6.51, 6.15, 6.02, 6.23 **21.** 12.3, 11.9, 11.09, 12.08

Find the mean, median, and mode(s) of the data. *(p. 918)*

22. Cost (in dollars) of computer monitors: 296, 215, 426, 390, 351, 215, 289

23. Size (in megabytes) of files to be downloaded: 10.9, 12.1, 6.4, 2.8, 5.1, 7.6

Evaluate the expression.

24. ${}_5P_3$ *(p. 851)* **25.** ${}_{10}P_4$ *(p. 851)* **26.** ${}_8C_4$ *(p. 856)* **27.** ${}_7C_2$ *(p. 856)*

13.6 Use Measures of Central Tendency and Dispersion

Before You analyzed surveys and samples.

Now You will compare measures of central tendency and dispersion.

Why? So you can analyze and compare data, as in Example 1.

Key Vocabulary
- measure of dispersion
- range
- mean absolute deviation

KEY CONCEPT *For Your Notebook*

Measures of Central Tendency

The **mean**, or *average*, of a numerical data set is denoted by $\bar{x}$, which is read as "x-bar." For the data set $x_1, x_2, \ldots, x_n$, the mean is $\bar{x} = \dfrac{x_1 + x_2 + \ldots + x_n}{n}$.

The **median** of a numerical data set is the middle number when the values are written in numerical order. If the data set has an even number of values, the median is the mean of the two middle values.

The **mode** of a data set is the value that occurs most frequently. There may be one mode, no mode, or more than one mode.

EXAMPLE 1 Compare measures of central tendency

The heights (in feet) of 8 waterfalls in the state of Washington are listed below. Which measure of central tendency best represents the data?

1000, 1000, 1181, 1191, 1200, 1268, 1328, 2584

Solution

$$\bar{x} = \frac{1000 + 1000 + 1181 + 1191 + 1200 + 1268 + 1328 + 2584}{8} = \frac{10{,}752}{8} = 1344$$

The median is the mean of the two middle values, 1191 and 1200, or 1195.5.

The mode is 1000.

▶ The median best represents the data. The mode is significantly less than most of the data, and the mean is significantly greater than most of the data.

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✓ GUIDED PRACTICE for Example 1

1. **WHAT IF?** In Example 1, suppose you eliminate the greatest data value, 2584. Which measure of central tendency best represents the remaining data? *Explain* your reasoning.

MEASURES OF DISPERSION A **measure of dispersion** describes the dispersion, or spread, of data. Two such measures are the *range*, which gives the length of the interval containing the data, and the *mean absolute deviation*, which gives the average variation of the data from the mean.

KEY CONCEPT *For Your Notebook*

Measures of Dispersion

The **range** of a numerical data set is the difference of the greatest value and the least value.

The **mean absolute deviation** of the data set $x_1, x_2, \ldots, x_n$ is given by:

$$\text{Mean absolute deviation} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \ldots + |x_n - \bar{x}|}{n}$$

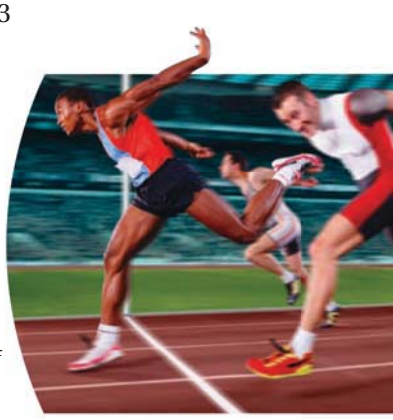
REVIEW ABSOLUTE VALUE
For help with absolute value, see p. 66.

EXAMPLE 2 Compare measures of dispersion

RUNNING The top 10 finishing times (in seconds) for runners in two men's races are given. The times in a 100 meter dash are in set *A*, and the times in a 200 meter dash are in set *B*. Compare the spread of the data for the two sets using (**a**) the range and (**b**) the mean absolute deviation.

A: 10.62, 10.94, 10.94, 10.98, 11.05, 11.13, 11.15, 11.28, 11.29, 11.32

B: 21.37, 21.40, 22.23, 22.23, 22.34, 22.34, 22.36, 22.60, 22.66, 22.73



Solution

a. *A*: $11.32 - 10.62 = 0.7$ *B*: $22.73 - 21.37 = 1.36$

▶ The range of set *B* is greater than the range of set *A*. So, the data in *B* cover a wider interval than the data in *A*.

b. The mean of set *A* is 11.07, so the mean absolute deviation is:

$$\frac{|10.62 - 11.07| + |10.94 - 11.07| + \ldots + |11.32 - 11.07|}{10} = 0.164$$

The mean of set *B* is 22.226, so the mean absolute deviation is:

$$\frac{|21.37 - 22.226| + |21.40 - 22.226| + \ldots + |22.73 - 22.226|}{10} = 0.3364$$

▶ The mean absolute deviation of set *B* is greater, so the average variation from the mean is greater for the data in *B* than for the data in *A*.

REVIEW NEGATIVE NUMBERS
When using the formula for mean absolute deviation, you will encounter negative numbers. For help with negative numbers, see p. 64.

✓ **GUIDED PRACTICE** for Example 2

2. RUNNING The top 10 finishing times (in seconds) for runners in a men's 400 meter dash are 46.89, 47.65, 48.15, 49.05, 49.19, 49.50, 49.68, 51.09, 53.31, and 53.68. *Compare* the spread of the data with that of set *A* in Example 2 using (**a**) the range and (**b**) the mean absolute deviation.

13.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7 and 19

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 17, 19, and 22

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The value that occurs most frequently in a data set is called the __?__ of the data.

2. ★ **WRITING** How are measures of central tendency and measures of dispersion used to compare data?

EXAMPLE 1 on p. 875 for Exs. 3–10

MEASURES OF CENTRAL TENDENCY **Find the mean, median, and mode(s) of the data.**

3. 1, 1, 1, 2, 3, 3, 5, 5, 6
4. 9, 10, 12, 15, 16
5. 13, 16, 19, 20, 22, 25, 30, 31
6. 14, 15, 15, 14, 14, 16, 18, 15
7. 5.52, 5.44, 3.60, 5.76, 3.80, 7.22
8. 300, 320, 341, 348, 360, 333

9. ★ **MULTIPLE CHOICE** What is the median of the data set?

0.7, 0.3, 0.7, 0.8, 0.9, 0.4, 1.0, 1.6, 1.2

Ⓐ 0.7 Ⓑ 0.8 Ⓒ 0.9 Ⓓ 1.0

10. **ERROR ANALYSIS** *Describe* and correct the error in finding the median of the data set.

7 4 6 2 4 6 8 8 3

The median is 4.

EXAMPLE 2 on p. 876 for Exs. 11–16

MEASURES OF DISPERSION **Find the range and mean absolute deviation of the data. Round to the nearest hundredth, if necessary.**

11. 30, 35, 20, 85, 60
12. 111, 135, 115, 120, 145, 130
13. 30, 45, 52, 48, 100, 45, 42, 45
14. 505, 510, 480, 550, 495, 500
15. 1.25, 1.50, 1.70, 0.85, 1.00, 1.25
16. 38.2, 80.1, 2.6, 84.2, 2.5, 5.5

17. ★ **WRITING** *Explain* why the mean absolute deviation of a data set is generally a better measure of dispersion than the range.

18. **CHALLENGE** Write a data set that has a mean of 10, a median of 10, and modes of 5 and 8.

PROBLEM SOLVING

EXAMPLE 1 on p. 875 for Exs. 19–20

19. ★ **SHORT RESPONSE** The weights (in pounds) of ten pumpkins are 22, 21, 24, 24, 5, 24, 5, 23, 24, and 24.

a. What is the range of the pumpkin weights?

b. Find the mean, median, and mode(s) of the pumpkin weights.

c. Which measure of central tendency best represents the data? *Explain*.

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20. POPULATION The population densities (in people per square mile) for each of the 10 most densely populated states in 2003 were 719.0, 418.5, 315.6, 563.6, 820.6, 1164.6, 406.5, 279.3, 275.9, and 1029.9.

a. Find the mean, median, and mode(s) of the data set.

b. Which measure of central tendency best represents the data? *Explain*.

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EXAMPLE 2 on p. 876 for Ex. 21

21. BOWLING The average scores of the bowlers on two different bowling teams are given. *Compare* the spreads of the data sets using (**a**) the range and (**b**) the mean absolute deviation.

Team 1: 162, 150, 173, 202 **Team 2:** 140, 153, 187, 196

22. ★ EXTENDED RESPONSE The Mississippi River discharges an average of 230 million tons of sediment per year. The average sediment discharges (in millions of tons per year) of the seven U.S. rivers with the greatest discharges are 230, 80, 65, 40, 25, 15, and 11. For parts (a)–(c) below, round your answers to the nearest whole number, if necessary.

a. Find the mean and median of the data. Which measure represents the data better? *Explain*.

b. Find the mean of the data for the other six rivers, excluding the Mississippi River. Does this mean represent the data better than the mean you found in part (a)? *Explain*.

c. Find the range and mean absolute deviation of the data for all seven rivers. *Describe* what the measures tell you about the dispersion of the data.

Fishing boat on the Mississippi River

23. CHALLENGE So far, you have scored 84, 92, 76, 88, and 76 on five of the six tests you will take in a particular class. Your goal is to finish the year with a test average of 85 or greater.

a. Let x represent your last test score. Write an expression for the mean of your test scores. Then write and solve an inequality to find the possible scores you can achieve in order to meet your goal.

b. After the last test, your teacher tells you that the median of your six test scores is 86. Can you tell whether you met your goal? *Explain*.

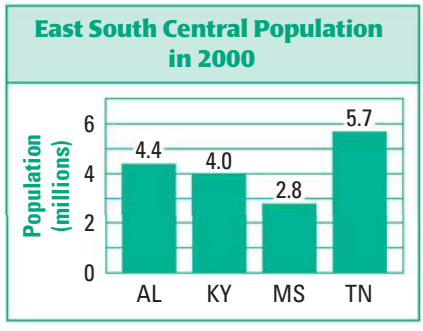
MIXED REVIEW

PREVIEW Prepare for Lesson 13.7 in Ex. 24.

24. POPULATION The bar graph shows the populations of Alabama, Kentucky, Mississippi, and Tennessee (the East South Central states) according to the U.S. Census of 2000. *(p. 933)*

a. What was the total population of the four states?

b. How much greater was the population of Tennessee than the population of Kentucky?



Extension

Use after Lesson 13.6

Calculate Variance and Standard Deviation

GOAL Find the variance and standard deviation of a data set.

Key Vocabulary
- **variance**
- **standard deviation**

In addition to range and mean absolute deviation, *variance* and *standard deviation* are also measures of dispersion that can be used to describe the spread of a set of data.

KEY CONCEPT *For Your Notebook*

Variance and Standard Deviation

The **variance** of a numerical data set is denoted by σ^2, which is read as "sigma squared." For the data set $x_1, x_2, \ldots, x_n$, the variance is given by:

$$\sigma^2 = \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_n - \overline{x})^2}{n}$$

The **standard deviation** of a numerical data set is denoted by σ, which is read as "sigma." For the data set $x_1, x_2, \ldots, x_n$, the standard deviation is the square root of the variance and is given by:

$$\sigma = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_n - \overline{x})^2}{n}}$$

EXAMPLE 1 Find variance and standard deviation

E-MAIL SIZES The sizes of e-mails (in kilobytes) in your inbox are 1, 2, 2, 7, 4, 1, 10, 3, and 6. Find the variance and standard deviation of the data.

Solution

STEP 1 **Find** the mean.

$$\overline{x} = \frac{1 + 2 + 2 + 7 + 4 + 1 + 10 + 3 + 6}{9} = \frac{36}{9} = 4$$

STEP 2 **Find** the variance.

$$\sigma^2 = \frac{(1 - 4)^2 + (2 - 4)^2 + \ldots + (6 - 4)^2}{9} = \frac{76}{9} = 8.444\ldots$$

STEP 3 **Find** the standard deviation.

$$\sigma = \sqrt{\sigma^2} = \sqrt{8.444\ldots} \approx 2.9$$

▶ The variance is about 8.4, and the standard deviation is about 2.9.

IMPROVE ACCURACY
The more accurate the value of σ^2 you use to calculate σ, the more accurate the value of σ you obtain. In the final answer, both results are rounded.

USING A CALCULATOR You can use a graphing calculator to find the standard deviation of a data set.

EXAMPLE 2 Find standard deviation

HOUSEHOLDS In 2000 the numbers (in thousands) of households in the 13 states with Atlantic Ocean coastline are given. Find the standard deviation of the data.

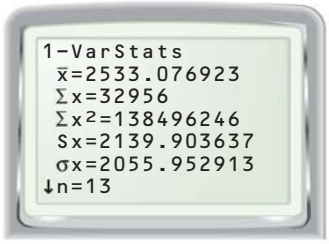
299 6338 3006 518 1981 2444 475 3065
7057 3132 408 1534 2699

Solution

STEP 1 **Enter** the data into a graphing calculator. Press STAT and select Edit. Enter the data into List 1 (L_1).

STEP 2 **Calculate** the standard deviation. Press STAT. From the CALC menu select 1-Var Stats.

On this screen, σ_x stands for standard deviation.



▶ The standard deviation of the data is about 2056.

PRACTICE

EXAMPLE 1 on p. 879 for Exs. 1–3

Use the formulas for variance and standard deviation to find the variance and standard deviation of the data. Round to the nearest tenth, if necessary.

1. 4, 5, 3, 2, 4, 7, 8, 9, 4, 6, 7, 8, 9, 1

2. 14, 16, 19, 20, 28, 7, 24, 15, 16, 30, 33, 24

3. 110, 205, 322, 608, 1100, 240, 185, 552, 418, 300

EXAMPLE 2 on p. 880 for Exs. 4–7

In Exercises 4–6, use a graphing calculator to find the standard deviation of the data. Round to the nearest tenth, if necessary.

4. 3.5, 3.8, 4.1, 3.0, 3.8, 3.6, 3.3, 4.0, 3.8, 3.9, 3.2, 3.0, 3.3, 4.2, 3.0

5. 66, 43, 9, 28, 7, 5, 90, 9, 78, 6, 69, 55, 28, 43, 10, 54, 13, 88, 21, 4

6. 1002, 1540, 480, 290, 2663, 3800, 690, 1301, 1750, 2222, 4040, 800

7. **REASONING** The heights (in feet) of 9 pecan trees are 72, 84, 81, 78, 80, 86, 70, 80, and 88. For parts (a)–(c) below, round your answers to the nearest tenth.

a. Find the standard deviation of the data.

b. Suppose you include a pecan tree with a height of 136 feet. *Predict* the effect of the additional data on the standard deviation of the data set.

c. Find the standard deviation of the new data set in part (b). *Compare* the results to your prediction in part (b).

13.7 Interpret Stem-and-Leaf Plots and Histograms

Before You found measures of central tendency and dispersion.

Now You will make stem-and-leaf plots and histograms.

Why? So you can analyze historical data, as in Ex. 20.

Key Vocabulary
- **stem-and-leaf plot**
- **frequency**
- **frequency table**
- **histogram**

A **stem-and-leaf plot** is a data display that organizes data based on their digits. Each value is separated into a *stem* (the leading digit(s)) and a *leaf* (the last digit). A stem-and-leaf plot has a key that tells you how to read the data. A stem-and-leaf plot shows how the data are distributed.

EXAMPLE 1 Make a stem-and-leaf plot

BASEBALL The number of home runs hit by the 20 baseball players with the best single-season batting averages in Major League Baseball since 1900 are listed below. Make a stem-and-leaf plot of the data.

14, 25, 8, 8, 7, 7, 19, 37, 39, 18, 42, 23, 4, 32, 14, 21, 3, 12, 19, 41

Solution

STEP 1 **Separate** the data into stems and leaves.

Home Runs

Stem	Leaves
0	8 8 7 7 4 3
1	4 9 8 4 2 9
2	5 3 1
3	7 9 2
4	2 1

Key: 1 | 4 = 14 home runs

STEP 2 **Write** the leaves in increasing order.

Home Runs

Stem	Leaves
0	3 4 7 7 8 8
1	2 4 4 8 9 9
2	1 3 5
3	2 7 9
4	1 2

Key: 1 | 4 = 14 home runs

INTERPRET INTERVALS
Each stem in a stem-and-leaf plot defines an interval. For instance, the stem 2 represents the interval 20–29. The data values in this interval are 21, 23, and 25.

✓ GUIDED PRACTICE for Example 1

1. **U.S. HISTORY** The years in which each of the first 20 states were admitted to the Union are listed below. Make a stem-and-leaf plot of the years.

   1788, 1787, 1788, 1816, 1792, 1812, 1788, 1788, 1817, 1788, 1787, 1788, 1789, 1803, 1787, 1790, 1788, 1796, 1791, 1788

2. **REASONING** In Example 1, describe the distribution of the data on the intervals represented by the stems. Are the data clustered together in a noticeable way? *Explain.*

EXAMPLE 2 Interpret a stem-and-leaf plot

GYMNASTICS The back-to-back stem-and-leaf plot shows the ages of members of the U.S men's and women's 2004 Olympic gymnastics teams. Compare the ages of the gymnasts on the two teams.

2004 Olympic Gymnast Ages

Men		Women
	1	6 6 8 8
7 4 3 1 1	2	5 6
0	3	

Key: 1 | 2 | 5 = 21, 25

Solution

Consider the distribution of the data. The interval for 10–19 years old contains more than half of the female gymnasts. The interval for 20–29 years old contains more than half of the male gymnasts. The clustering of the data shows that the men's team was generally older than the women's team.

FREQUENCY The **frequency** of an interval is the number of data values in that interval. A stem-and-leaf plot shows the frequencies of intervals determined by the stems. A **frequency table** is also used to group data values into equal intervals, with no gaps between intervals and no intervals overlapping.

A **histogram** is a bar graph that displays data from a frequency table. Each bar represents an interval. Because intervals have equal size, the bars have equal width. A bar's length indicates the frequency. There is no space between bars.

EXAMPLE 3 Make a histogram

SANDWICH PRICES The prices (in dollars) of sandwiches at a restaurant are listed below. Make a histogram of the data.

4.00, 4.00, 4.25, 4.50, 4.75, 4.25, 5.95, 5.50, 5.50, 5.75

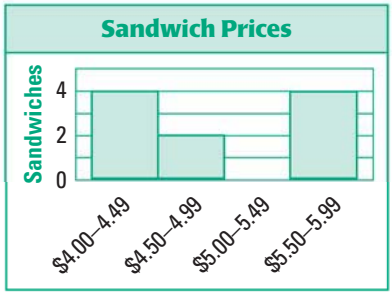
Solution

CHOOSE AN INTERVAL SIZE
To choose the interval size for a frequency table, divide the range of the data by the number of intervals you want the table to have. Use the quotient as an approximate interval size.

STEP 1 **Choose** intervals of equal size that cover all of the data values. Organize the data using a frequency table.

Prices	Sandwiches
\$4.00–4.49	IIII
\$4.50–4.99	II
\$5.00–5.49	
\$5.50–5.99	IIII

STEP 2 **Draw** the bars of the histogram using the intervals from the frequency table.



✓ GUIDED PRACTICE for Examples 2 and 3

3. TELEVISION The back-to-back stem-and-leaf plot shows the percents of students in 24 countries who report watching television for 4 or more hours each day. *Compare* the data for female and male students.

Female		Male
9 9 9 8 8 8 6 6 5 4	1	7 8 9
6 6 5 4 3 3 0 0	2	0 1 2 2 4 5 5 6 6 7 7 8
8 3 2 2 1	3	4 6 6 8 8 9
1	4	0 6 6

Key: 4 | 1 | 7 = 14%, 17%

4. PRECIPITATION The average number of days each month with precipitation of 0.01 inch or more in Buffalo, New York, are 20, 17, 16, 14, 13, 11, 10, 10, 11, 12, 16, and 19. Make a histogram of the data.

13.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3 and 19

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 8, 9, 15, and 20

SKILL PRACTICE

1. VOCABULARY Copy and complete: The number of data values in an interval is the __?__ of that interval.

2. ★ WRITING *Explain* how a histogram differs from a bar graph.

EXAMPLE 1 on p. 881 for Exs. 3–7

STEM-AND-LEAF PLOTS Make a stem-and-leaf plot of the data.

3. 17, 31, 42, 33, 38, 20, 24, 30, 39, 38, 35, 20, 55

4. 2, 8, 17, 7, 14, 20, 32, 5, 33, 6, 6, 8, 11, 9

5. 121, 124, 133, 111, 109, 182, 105, 127, 156, 179, 142

6. 1.23, 1.05, 1.11, 1.29, 1.31, 1.19, 1.45, 1.22, 1.19, 1.35

7. ERROR ANALYSIS *Describe* and correct the error in making a stem-and-leaf plot of the following data: 18, 19, 18, 19, 20, 20, 21, 22, 18, 19, 20, 21, 23, 21.

1	888999
2	00011123

STEM-AND-LEAF PLOT In Exercises 8 and 9, consider the back-to-back stem-and-leaf plot that shows data sets *A* and *B*.

Set *A*		Set *B*
1 1 1	2	
4 3 3 2	3	1 2 2
2 0	4	1 1 3 4
	5	0 1

Key: 2 | 3 | 1 = 32, 31

8. ★ MULTIPLE CHOICE What is the median of data set *A*?

Ⓐ 21 Ⓑ 32 Ⓒ 33 Ⓓ 34

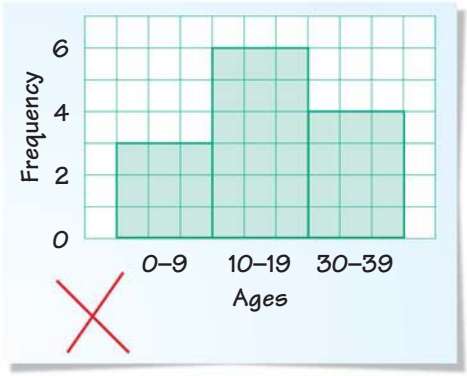
9. ★ MULTIPLE CHOICE What is the range of data set *B*?

Ⓐ 18 Ⓑ 19 Ⓒ 20 Ⓓ 21

EXAMPLE 3 on p. 882 for Exs. 10–14

10. ERROR ANALYSIS *Describe* and correct the error in creating a histogram using the frequency table below.

Ages	0–9	10–19	20–29	30–39
Frequency	III	~~IIII~~ I		IIII



HISTOGRAMS **Make a histogram of the data.**

11. 55, 82, 94, 75, 61, 69, 77, 98, 81, 83, 75, 90, 51

12. 12, 0, 22, 31, 14, 7, 7, 45, 31, 28, 21, 25, 25, 18

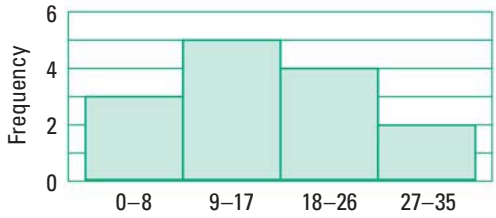
13. 0.01, 0.13, 0.09, 1.10, 1.33, 0.99, 0.50, 0.95, 1.05, 1.50, 0.75, 1.01

14. 111, 109, 224, 657, 284, 120, 119, 415, 180, 105, 208, 108

15. ★ **WRITING** *Explain* why a histogram can show the distribution of the data below better than a stem-and-leaf plot.

15, 21, 18, 10, 12, 11, 17, 18, 16, 12, 20, 12, 17, 16

16. CHALLENGE Create a stem-and-leaf plot that has the same distribution of data as the histogram shown. *Explain* the steps you took to create the stem-and-leaf plot.



PROBLEM SOLVING

EXAMPLE 1 on p. 881 for Ex. 17

17. HEIGHTS The heights (in inches) of players on a boys' basketball team are as follows: 80, 76, 81, 69, 81, 78, 74, 68, 78, 74, 81, 72, 69, 81, 70. Make a stem-and-leaf plot of the heights.

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EXAMPLE 3 on p. 882 for Exs. 18–19

18. SURVEY A survey asked people how many 8 ounce glasses of water they drink in one day. The results are below. Make a histogram of the data.

3, 0, 9, 1, 4, 2, 11, 5, 3, 6, 0, 5, 7, 8, 5, 2, 9, 6, 10, 2, 4

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19. MEMORY A survey asked people how many phone numbers they have memorized. The results are shown in the table.

Phone numbers	1–5	6–10	11–15	16–20	21–25
Frequency	88	85	50	28	14

a. Make a histogram of the data.

b. What is the probability that a person surveyed, chosen at random, has 11–25 phone numbers memorized?

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

EXAMPLE 2 on p. 882 for Ex. 20

20. ★ **EXTENDED RESPONSE** The back-to-back stem-and-leaf plot shows the numbers of days the House of Representatives and the Senate spent in session each year from 1996 to 2004.

a. What was the median number of days the House of Representatives spent in session? the Senate?

b. What is the range of the number of days the House of Representatives spent in session? the Senate?

c. *Compare* the data for the House of Representatives and the Senate. What does the distribution of the data tell you?

Days in Session

House		Senate
9 0	11	
3 2	12	
7 5 3 2	13	2 3
2	14	1 3 9
	15	3
	16	2 7
	17	3

Key: 2 | 14 | 1 = 142, 141

21. MAYFLOWER The known ages (in years) of adult male passengers on the *Mayflower* at the time of its departure are listed below.

21, 34, 29, 38, 30, 54, 39, 20, 35, 64, 37, 45, 21, 25, 55, 45, 40, 38, 38, 21, 21, 20, 34, 38, 50, 41, 48, 18, 32, 21, 32, 49, 30, 42, 30, 25, 38, 25, 20

a. Make a stem-and-leaf plot of the ages.

b. Find the median age and range of the ages.

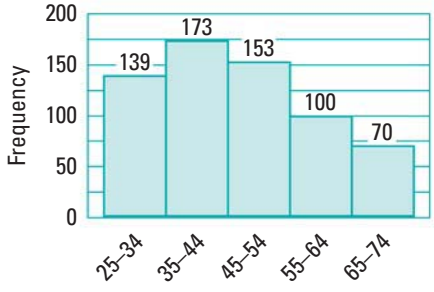
c. According to one source, the age of passenger Thomas English was unknown at the time of the *Mayflower's* departure. What is the probability that he was 18–29 years old? *Explain* your reasoning.

Replica of the *Mayflower*

22. CHALLENGE Refer to the histogram shown.

a. Find the midpoint of each interval. Multiply each midpoint by the frequency of its interval. Add these products. Divide the sum by the sum of all the frequencies.

b. Does the your final result in part (a) best approximate the mean, the median, or the mode of the data? *Explain* your answer.



MIXED REVIEW

PREVIEW Prepare for Lesson 13.8 in Exs. 23–25.

Find the mean, median, and mode(s) of the data. *(p. 918)*

23. Ages of family members (in years): 62, 35, 51, 28, 22, 25, 16, 58, 30, 14

24. Minutes of exercise each day: 35, 20, 25, 20, 0, 30, 45, 40, 20, 30, 35, 0

25. Hours worked per week: 10, 9, 11, 12, 8, 15, 20, 9, 16, 14, 15, 12

Tell whether the question is potentially biased. *Explain* your answer. If the question is biased, rewrite it so that it is not. *(p. 871)*

26. Don't you agree that science fiction movies are boring?

27. Is your ideal vacation a trip to Florida or a trip to Alaska?

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Keystrokes

13.7 Draw Histograms

QUESTION How can you use a graphing calculator to make a histogram?

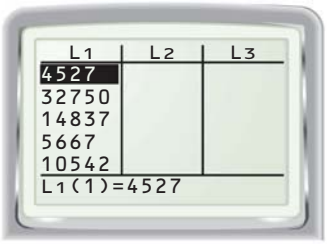
EXAMPLE Make a histogram

POPULATION The populations (in thousands) of metropolitan areas in the states with the greatest metropolitan populations in the United States in 2000 are listed below. Make a histogram of the data.

4527 32,750 14,837 5667 10,542 4390 4911 6101 8169 3795 8414
17,473 5437 9214 10,392 3862 17,692 5528 4899 3640

STEP 1 *Enter the data*

Go to the STAT menu and choose Edit. Enter the data into List 1.

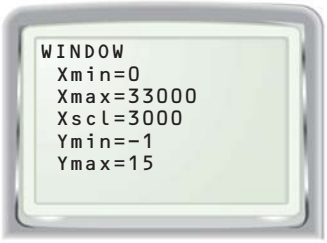


STEP 2 *Select histogram*

Go to the STAT PLOT screen. Select Plot 1. Use the settings shown below.

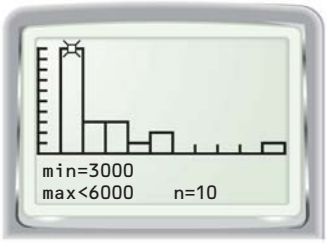
STEP 3 *Set the viewing window*

Go to the WINDOW screen. Use the settings shown below.



STEP 4 *Graph*

Press GRAPH. Use the *trace* feature to move from bar to bar.



DRAW CONCLUSIONS

1. *Describe* the distribution of the population data in the example above.
2. **BOWLING** Use a graphing calculator to make a histogram of the following bowling scores: 200, 210, 105, 300, 180, 175, 162, 110, 140, 300, 152, 165, 175, 115, 250, 270, 145, 182, 164, 122, 141, 135, 189, 170, 151, 158.

13.8 Interpret Box-and-Whisker Plots

Before You made stem-and-leaf plots and histograms.

Now You will make and interpret box-and-whisker plots.

Why? So you can compare sets of scientific data, as in Ex. 19.

Key Vocabulary
- box-and-whisker plot
- quartile
- interquartile range
- outlier

A **box-and-whisker plot** organizes data values into four groups. Ordered data are divided into lower and upper halves by the median. The median of the lower half is the **lower quartile**. The median of the upper half is the **upper quartile**.

EXAMPLE 1 Make a box-and-whisker plot

SONG LENGTHS The lengths of songs (in seconds) on a CD are listed below. Make a box-and-whisker plot of the song lengths.

173, 206, 179, 257, 198, 251, 239, 246, 295, 181, 261

Solution

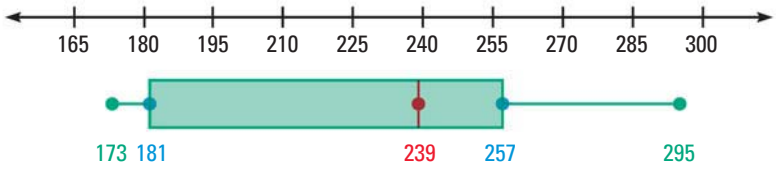
STEP 1 **Order** the data. Then find the median and the quartiles.

Lower half — Median — Upper half

173, 179, **181**, 198, 206, **239**, 246, 251, **257**, 261, 295

Lower quartile — Upper quartile

STEP 2 **Plot** the median, the quartiles, the maximum value, and the minimum value below a number line.



STEP 3 **Draw** a box from the lower quartile to the upper quartile. Draw a vertical line through the median. Draw a line segment (a "whisker") from the box to the maximum and another from the box to the minimum.

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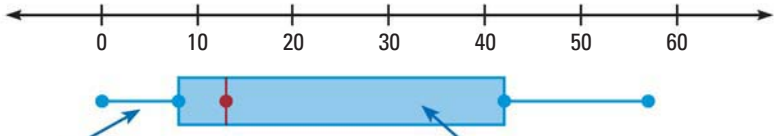
✓ GUIDED PRACTICE for Example 1

1. Make a box-and-whisker plot of the ages of eight family members: 60, 15, 25, 20, 55, 70, 40, 30.

INTERPRET A BOX-AND-WHISKER PLOT A box-and-whisker plot separates data into four groups: the two parts of the box and the two whiskers. Each part contains approximately the same number of data values.



Each whisker represents about 25% of the data.

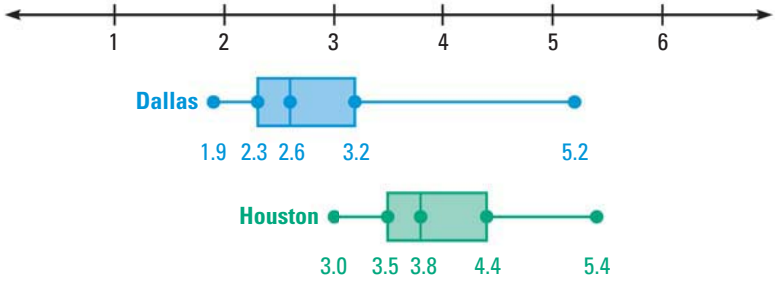
The box on each side of the median represents about 25% of the data.

INTERPRET VARIATION
The interquartile range measures the variation in the middle half of the data and ignores the extreme values, whose variation may not be representative of the data.

You know that the range of a data set is the difference of the maximum value and the minimum value. The **interquartile range** of a data set is the difference of the upper quartile and the lower quartile.

EXAMPLE 2 Interpret a box-and-whisker plot

PRECIPITATION The box-and-whisker plots below show the normal precipitation (in inches) each month in Dallas and in Houston, Texas.



a. For how many months is Houston's precipitation less than 3.5 inches?

b. Compare the precipitation in Dallas with the precipitation in Houston.

Solution

a. For Houston, the lower quartile is 3.5. A whisker represents 25% of the data, so for 25% of 12 months, or 3 months, Houston has less than 3.5 inches of precipitation.

b. The median precipitation for a month in Dallas is 2.6 inches. The median for Houston is 3.8 inches. In general, Houston has more precipitation.

For Dallas, the interquartile range is $3.2 - 2.3$, or 0.9 inch. For Houston, the interquartile range is $4.4 - 3.5 = 0.9$ inch. So, the cities have the same variation in the middle 50% of the data. The range for Dallas is greater than the range for Houson. When all the data are considered, Dallas has more variation in precipitation.

INTERPRET QUARTILES
When the number of data values is a multiple of 4, the median and quartiles will divide the data into four groups of *exactly* the same size.

✓ GUIDED PRACTICE for Example 2

2. PRECIPITATION In Example 2, for how many months was the precipitation in Dallas more than 2.6 inches?

OUTLIERS A value that is widely separated from the rest of the data in a data set is called an **outlier**. Typically, a data value is considered to be an outlier if it is greater than the upper quartile by more than 1.5 times the interquartile range or if it is less than the lower quartile by more than 1.5 times the interquartile range.

★ EXAMPLE 3 Standardized Test Practice

The normal monthly amounts of precipitation (in inches) in Dallas are: 1.9, 2.4, 3.1, 3.2, 5.2, 3.2, 2.1, 2.0, 2.4, 4.1, 2.6, 2.6. These data were used to create the box-and-whisker plot in Example 2. Which value, if any, is an outlier?

Ⓐ 1.9 Ⓑ 5.2 Ⓒ 1.9 and 5.2 Ⓓ No outlier

Solution

From Example 2, you know the interquartile range of the data is 0.9 inch. Find 1.5 times the interquartile range: $1.5(0.9) = 1.35$.

From Example 2, you also know that the lower quartile is 2.3 and the upper quartile is 3.2. A value less than $2.3 - 1.35 = 0.95$ is an outlier. A value greater than $3.2 + 1.35 = 4.55$, is an outlier. Notice that $5.2 > 4.55$.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 3

3. Which value, if any, is an outlier in the data set?

3.7, 3.0, 3.4, 3.6, 5.2, 5.4, 3.2, 3.8, 4.3, 4.5, 4.2, 3.7

Ⓐ 3.0 Ⓑ 5.4 Ⓒ 3.0 and 5.4 Ⓓ No outlier

13.8 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3 and 17

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 8, 9, 18, and 19

SKILL PRACTICE

1. VOCABULARY What is the interquartile range of a data set?

2. ★ WRITING *Explain* how you can identify an outlier in a data set.

EXAMPLE 1 on p. 887 for Exs. 3–7

BOX-AND-WHISKER PLOTS **Make a box-and-whisker plot of the data.**

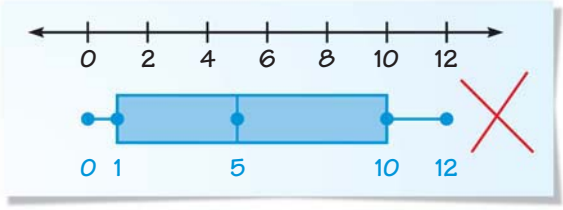
3. 1, 7, 0, 7, 2, 6, 3, 6, 0, 7, 8

4. 10, 1, 7, 5, 1, 8, 5, 4, 6, 5, 9, 12

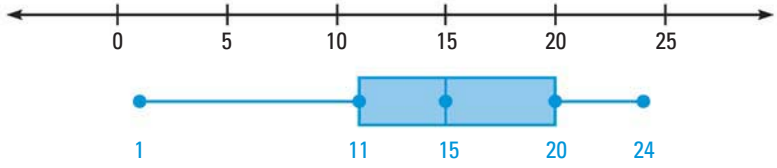
5. 52, 20, 24, 45, 35, 32, 39, 42, 23, 64

6. 0.8, 0.4, 0.3, 0.6, 0.7, 0.2, 0.7, 0.9

7. **ERROR ANALYSIS** *Describe* and correct the error in creating a box-and-whisker plot of the data 0, 2, 4, 0, 6, 10, 8, 12, 5.



BOX-AND-WHISKER PLOT **In Exercises 8–10, use the box-and-whisker plot.**



EXAMPLE 2 on p. 888 for Exs. 8–10

8. ★ **MULTIPLE CHOICE** About what percent of the data are greater than 20?

   (A) 25% (B) 50% (C) 75% (D) 100%

9. ★ **MULTIPLE CHOICE** About what percent of the data are less than 15?

   (A) 25% (B) 50% (C) 75% (D) 100%

10. **ERROR ANALYSIS** *Describe* and correct the error in interpreting the box-and-whisker plot.

About 25% of the data values lie between 11 and 20.

EXAMPLES 1 and 3 on pp. 887, 889 for Exs. 11–13

OUTLIERS **Make a box-and-whisker plot of the data. Identify any outliers.**

11. Hours worked per week: 15, 15, 10, 12, 22, 10, 8, 14, 18, 22, 18, 15, 12, 11, 10
12. Prices of MP3 players: $124, $95, $105, $110, $95, $124, $300, $190, $114
13. Annual salaries: $30,000, $35,000, $48,000, $68,500, $32,000, $38,000

14. **CHALLENGE** Two data sets have the same mean, the same interquartile range, and the same range. Is it possible for the box-and-whisker plots of such data sets to be different? *Justify* your answer by creating data sets that fit the situation.

PROBLEM SOLVING

EXAMPLE 1 on p. 887 for Exs. 15–16

15. **SEAWAY** The average sailing times to the Atlantic Ocean from several ports on the St. Lawrence Seaway are shown on the map. Make a box-and-whisker plot of the sailing times.

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○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

16. BASEBALL STATISTICS In 2004, Ichiro Suzuki scored 101 runs. The numbers of runs he scored against different opposing teams are listed below. Make a box-and-whisker plot of the numbers of runs scored.

Runs scored: 18, 8, 4, 8, 2, 8, 0, 9, 0, 4, 2, 5, 9, 1, 2, 1, 2, 11, 7

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EXAMPLES 1 and 3 on pp. 887, 889 for Exs. 17–18

17. RETAIL SALES The retail sales (in billions of dollars) of the nine U.S. states with the highest retail sales in 2002 are listed below.

California: $153.1	Florida: $118.2	Georgia: $38.4
Illinois: $52.4	New Jersey: $35.8	New York: $54.7
Ohio: $50.7	Pennsylvania: $49.9	Texas: $107.0

a. Make a box-and-whisker plot of the retail sales.

b. Which states, if any, had retail sales in 2002 that can be considered outliers?

18. ★ SHORT RESPONSE The stem-and-leaf plot shows the ages of the first 43 presidents of the United States when they first took the oath of office.

4	2 3 6 6 7 8 9 9
5	0 0 1 1 1 1 2 2 4 4 4 4 4 5 5 5 5 6 6 6 7 7 7 7 8
6	0 1 1 1 2 4 4 5 8 9

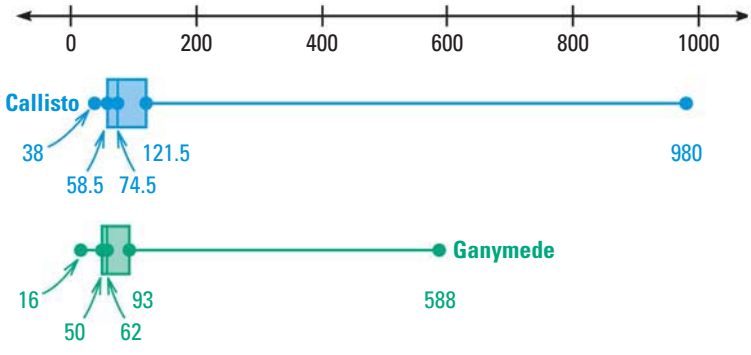
Key: 4 | 2 = 42 years

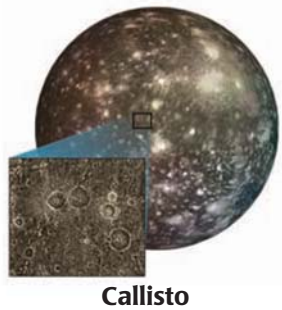
a. Make a box-and-whisker plot of the ages.

b. Ronald Reagan was the oldest United States president, and Theodore Roosevelt was the youngest. Can either of these presidents' ages be considered outliers? *Explain* why or why not.

EXAMPLE 2 on p. 888 for Ex. 19

19. ★ EXTENDED RESPONSE The box-and-whisker plots show the diameters (in kilometers) of craters on Jupiter's moons Callisto and Ganymede.





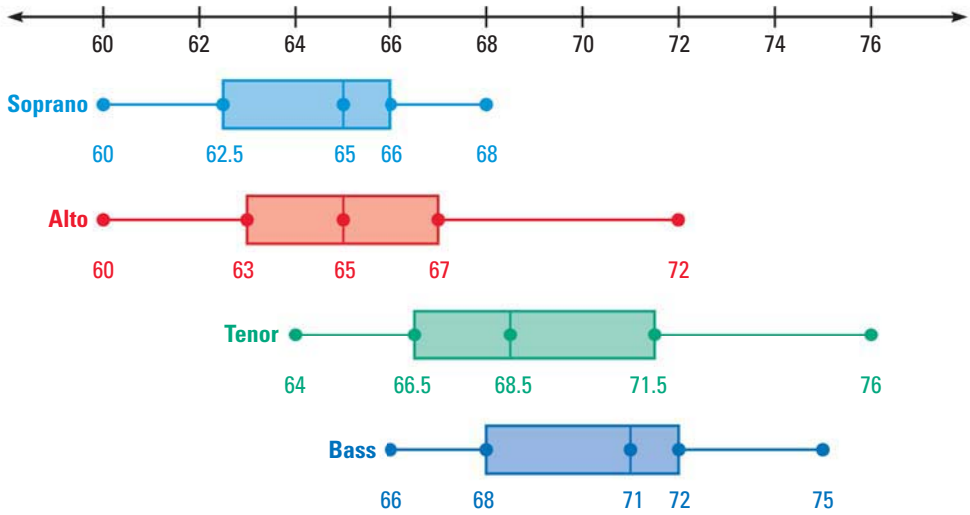
Callisto

a. *Compare* the diameters of craters on Callisto with the diameters of craters on Ganymede.

b. The largest crater in the United States is the Chesapeake Bay in Virginia, with a diameter of 90 kilometers. *Compare* the diameter of the Chesapeake Bay with diameters of craters on Callisto and Ganymede.

c. The largest crater on Earth is Vredefort in South Africa, with a diameter of 300 kilometers. *Compare* the diameter of Vredefort with the diameter of craters on Callisto and Ganymede.

20. **CHALLENGE** The box-and-whisker plots show the heights (in inches) of singers in a chorus, according to their voice parts. A soprano part has the highest pitch, followed by alto, tenor, and bass, respectively. Draw a conclusion about voice parts and heights. *Justify* your conclusion.



MIXED REVIEW

Make a stem-and-leaf plot of the data. *(p. 881)*

21. 56, 55, 54, 57, 28, 28, 53, 52, 56, 28, 25, 23, 17, 51, 54, 23, 20, 10

22. 71, 60, 39, 43, 81, 32, 33, 41, 37, 34, 51, 41, 32, 34, 48, 35, 36, 58

Make a histogram of the data. *(p. 881)*

23. 1.24, 2.45, 1.11, 2.09, 2.19, 1.99, 1.75, 1.65, 2.10, 2.30

24. 1.5, 5.12, 7.5, 7.1, 7.14, 9.7, 10.24, 1.3, 1.6, 1.6, 3.3, 3.12

QUIZ *for Lessons 13.5–13.8*

1. **HOTEL SURVEY** A hotel manager leaves guest comment cards in each room. Identify the population and classify the sampling method. *(p. 871)*

In Exercises 2 and 3, find the range and mean absolute deviation of the data. Round to the nearest hundredth, if necessary. *(p. 875)*

2. 62, 63, 70, 40, 50, 60

3. 14, 18, 22, 14, 14, 6, 17

4. Make a histogram of the data: 44, 52, 60, 47, 65, 40, 49, 45, 32, 68, 39. *(p. 881)*

5. Make a stem-and-leaf plot of the data: 1.8, 2.2, 1.2, 2.8, 3.6, 3.3, 1.8, 2.2. *(p. 881)*

6. **TEST SCORES** The scores on a math exam are given below. Make a box-and-whisker plot of the data. Identify any outliers. *(p. 887)*

76, 55, 88, 92, 79, 85, 90, 88, 85, 92, 100, 91, 90, 86, 88

EXTRA PRACTICE for Lesson 13.8, p. 950 **ONLINE QUIZ** at classzone.com

Graphing Calculator ACTIVITY *Use after Lesson 13.8*

@HomeTutor classzone.com Keystrokes

13.8 Draw Box-and-Whisker Plots

QUESTION How can you use a graphing calculator to make a box-and-whisker plot?

EXAMPLE Make a box-and-whisker plot

REPTILE SPECIES The number of known reptile species per 10,000 square kilometers in the countries of Asia (excluding the Middle East) and of Central America, South America, and the Caribbean are listed below. Make box-and-whisker plots of the numbers of species.

Asia: 36, 26, 49, 11, 32, 35, 27, 58, 91, 26, 8, 8, 12, 12, 23, 110, 4, 51, 41, 41, 62, 350, 77, 18, 81, 23, 18, 59

Central America, South America, and the Caribbean: 81, 125, 47, 69, 57, 107, 77, 73, 35, 123, 69, 116, 87, 37, 45, 53, 20, 124, 126, 35, 73, 60, 64

STEP 1 *Enter the data*

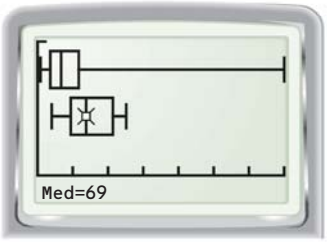
Enter the data for Asia into List 1. Enter the data for Central America, South America, and the Caribbean into List 2.

STEP 2 *Select box-and-whisker plot*

Go to the STAT PLOT screen and select the box-and-whisker plot for both Plot 1 and Plot 2. The Xlist for Plot 1 should be L_1, so that it displays the data from List 1. The Xlist for Plot 2 should be L_2, so that it displays the data from List 2. Make sure both plots are on.

STEP 3 *Set the viewing window*

Press ZOOM 9 to set the window so that it shows all of the data.



STEP 4 *Graph*

Press GRAPH. Use the trace feature to examine the box-and-whisker plots more closely. Notice that the graphing calculator refers to the lower quartile as Q_1 and the upper quartile as Q_3.

DRAW CONCLUSIONS

1. **REPTILE SPECIES** *Compare* the number of reptile species per 10,000 square kilometers in the countries of Central America, South America, and the Caribbean with the number in Asia.

2. **BIRD SPECIES** The number of threatened bird species per 10,000 square kilometers in the countries of two regions are listed below. Make box-and-whisker plots of the data and compare the data for the two regions.

   Middle East and Northern Africa: 13, 8, 11, 14, 12, 8, 4, 3, 5, 2, 11, 5, 11, 7, 6, 14, 4, 13

   North and South America: 5, 50, 41, 27, 103, 18, 64, 53, 3, 26, 64, 2, 11, 22

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 13.5–13.8

1. **MULTI-STEP PROBLEM** The ages of people who attended an opening reception for a theater production are listed below.

   54, 25, 28, 64, 30, 42, 33, 50, 27, 35, 40, 39, 41, 52, 49, 48, 56, 60, 58, 37, 56, 45, 57, 62

   a. Make a frequency table of the data.

   b. Make a histogram of the data.

2. **MULTI-STEP PROBLEM** A doctor would like to extend her office hours to better accommodate her patients. She asks each patient who visits her office on Tuesday which day the patient thinks the hours should be extended.

   a. Identify the population and classify the sampling method.

   b. Tell whether the survey method used is likely to result in a biased sample.

3. **GRIDDED ANSWER** The average lengths (in hours) of several morning commutes are listed below. How many minutes is the mean commute?

   0.25, 0.20, 0.50, 0.50, 0.50, 0.05, 0.65, 1.00, 1.50, 0.75, 0.50, 1.10, 0.60, 0.80, 1.00, 0.10

4. **EXTENDED RESPONSE** The prices (in dollars) of portable DVD players at two different stores are listed below.

   Store A: 280, 200, 260, 230, 200, 150, 300, 260, 500, 190

   Store B: 350, 190, 230, 250, 400, 200, 200, 220, 185, 150

   a. Find the mean, median, and mode(s) of each data set. Which measure of central tendency best represents each data set? *Explain* your reasoning.

   b. Find the range and mean absolute deviation of each data set. Which store's prices are more spread out? *Explain*.

   c. Can any of the prices of the portable DVD players be considered outliers? *Explain* your reasoning.

5. **OPEN-ENDED** A clothing store sells several different styles of jeans. The mean price of the jeans is \$27. The median price of the jeans is \$27.50. The mode of the prices is \$20. Make a list of prices of jeans that has these measures of central tendency.

6. **SHORT RESPONSE** The back-to-back stem-and-leaf plot below shows the lengths (in meters) of the eight best men's and women's final long jump results from the 2004 Olympics. *Compare* the lengths of the jumps by men with those by women.

Lengths (in meters) of Long Jump

Men		Women
	6	7 8 8 9
	7	0 0 0 1
6 5 3 3 2 2 2 0	8	

Key: 0 | 7 | 1 = 7.0 m, 7.1 m

7. **SHORT RESPONSE** The stem-and-leaf plot shows the number of games lost by 15 NCAA football coaches with the greatest career winning percentages after at least 10 years of coaching.

1	1 2 3 6 7 7 8
2	1 3 3 4 5 9
3	6
4	9

Key: 2 | 1 = 21 games

   a. Make a box-and-whisker plot of the data.

   b. Tom Osborne had a winning percentage of 83.6% over his career and lost 49 games. Can the number of games lost by Tom Osborne be considerred an outlier? *Explain* your reasoning.

13 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

Big Idea 1 Finding Probabilities of Simple and Compound Events

To find $P(A)$ when...	
all outcomes are equally likely, use $P(A) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$	you perform an experiment, use $P(A) = \frac{\text{Number of successes}}{\text{Number of trials}}$

To find $P(A \text{ or } B)$ when...	...use this formula
events A and B have no common outcomes	$P(A \text{ or } B) = P(A) + P(B)$
events A and B have at least one common outcome	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

To find $P(A \text{ and } B)$ when...	...use this formula
events A and B are independent	$P(A \text{ and } B) = P(A) \cdot P(B)$
events A and B are dependent	$P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$

Big Idea 2 Analyzing Sets of Data

You can find values that represent a typical data value using the following measures of central tendency:

mean, median, and mode

You can find values that describe the spread of data using the following measures of dispersion:

range, mean absolute deviation, and interquartile range

Big Idea 3 Making and Interpreting Data Displays

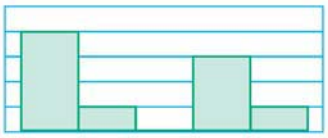
Use an appropriate display to show the distribution of a set of numerical data.

A **stem-and-leaf plot** organizes data based on their digits.

Stem	Leaves
1	0 1 1 2 3
2	0 0 0 2

Key: 1 | 0 = 10

A **histogram** shows the frequency of data on intervals of equal size, with no gaps or overlaps.



A **box-and-whisker plot** organizes data into four groups of approximately equal size.

13 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary Practice

REVIEW KEY VOCABULARY

- outcome, event, *p. 843*
- sample space, *p. 843*
- probability of an event, *p. 843*
- theoretical, experimental probability, *p. 844*
- odds in favor, odds against, *p. 845*
- permutation, *p. 851*
- *n* factorial, *p. 852*
- combination, *p. 856*
- compound event, *p. 861*
- mutually exclusive events, *p. 861*
- overlapping events, *p. 861*
- independent events, *p. 862*
- dependent events, *p. 862*
- survey, *p. 871*
- population, *p. 871*
- sample: random, stratified random, systematic, convenience, self-selected, *p. 871*
- biased sample, *p. 872*
- biased question, *p. 872*
- mean, median, mode, *p. 875*
- measure of dispersion, *p. 876*
- range, *p. 876*
- mean absolute deviation, *p. 876*
- stem-and-leaf plot, *p. 881*
- frequency, frequency table, *p. 882*
- histogram, *p. 882*
- box-and-whisker plot, *p. 887*
- lower quartile, upper quartile, *p. 887*
- interquartile range, *p. 888*
- outlier, *p. 889*

VOCABULARY EXERCISES

Copy and complete the statement.

1. An event that combines two or more events is a(n) _?_.
2. A possible result of an experiment is a(n) _?_.
3. **WRITING** *Compare* theoretical probability and experimental probability.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 13.

13.1 Find Probabilities and Odds *pp. 843–848*

EXAMPLE

A bag contains 15 red checkers and 15 black checkers. You choose a checker at random. Find the probability that you choose a black checker.

$$P(\text{black checker}) = \frac{\text{Number of black checkers}}{\text{Total number of checkers}} = \frac{15}{30} = \frac{1}{2}$$

EXERCISES

EXAMPLE 2 on p. 844 for Exs. 4–5

4. **CHECKERS** In the example above, suppose an extra red checker is added to the bag. Find the probability of randomly choosing a black checker.
5. **BAG OF LETTERS** A bag contains tiles. Each tile has one letter from the word HAPPINESS on it. You choose a tile at random. What is the probability that you choose a tile with the letter S?

13.2 Find Probabilities Using Permutations

pp. 851–855

EXAMPLE

You need to enter a 4 digit code in order to enter the building where you work. The digits are 4 different numbers from 1 to 5. You forgot the code and try to guess it. Find the probability that you guess correctly.

STEP 1 **Write** the number of possible outcomes as the number of permutations of 4 out of the 5 possible digits. This is ${}_5P_4$.

$${}_5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

STEP 2 **Find** the probability. Because only one of the permutations is the correct code, the probability that you guess the correct code is $\frac{1}{120}$.

EXERCISES

EXAMPLE 2
on p. 852
for Exs. 6–10

Evaluate the expression.

6. ${}_7P_6$ **7.** ${}_6P_2$ **8.** ${}_8P_5$ **9.** ${}_{13}P_{10}$

10. MUSIC You downloaded 6 songs. You randomly choose 4 of these songs to play. Find the probability that you play the first 4 songs you downloaded in the order in which you downloaded them.

13.3 Find Probabilities Using Combinations

pp. 856–859

EXAMPLE

For your government class, you must choose 3 states in the United States to research. You may choose your states from the 6 New England states. How many combinations of states are possible?

The order in which you choose the states is not important. So, to find the number of combinations of 6 states taken 3 at a time, find ${}_6C_3$.

$${}_6C_3 = \frac{6!}{(6-3)! \cdot 3!}$$ **Combinations formula**

$$= \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!} \cdot (3 \cdot 2 \cdot 1)}$$ **Expand factorials. Divide out common factor, 3!.**

$$= 20$$ **Simplify.**

EXERCISES

EXAMPLE 2
on p. 857
for Exs. 11–15

Evaluate the expression.

11. ${}_7C_6$ **12.** ${}_6C_2$ **13.** ${}_8C_5$ **14.** ${}_{13}C_{10}$

15. TICKETS You win 5 tickets to a concert. In how many ways can you choose 4 friends out of a group of 9 to take with you to the concert?

13.4 Find Probabilities of Compound Events

pp. 861–867

EXAMPLE

The sections of the spinner shown all have the same area. You spin the spinner. Find the probability that the spinner stops on red or on an even number.

Because 24 is an even number on a red section, stopping on red and stopping on an even number are overlapping events.

$$P(\text{red or even}) = P(\text{red}) + P(\text{even}) - P(\text{red and even})$$

$$= \frac{3}{8} + \frac{3}{8} - \frac{1}{8}$$

$$= \frac{5}{8}$$

EXERCISES

EXAMPLES 1 and 2 on pp. 861–862 for Exs. 16–19

You spin the spinner shown above. Find the specified probability.

16. P(green or odd)

17. P(blue or prime number)

18. P(blue or even)

19. P(red or multiple of 3)

EXAMPLE

A bag contains 5 red marbles, 3 blue marbles, 6 white marbles, and 2 green marbles. You choose one marble at random, put the marble aside, then choose a second marble at random. What is the probability that both marbles are blue?

Because you do not replace the first marble, the events are dependent. Before you choose a marble, there are 16 marbles, and 3 of them are blue. After you choose a blue marble, there are 2 blue marbles among 15 marbles left.

$$P(\text{blue and then blue}) = P(\text{blue}) \cdot P(\text{blue given blue})$$

$$= \frac{3}{16} \cdot \frac{2}{15}$$

$$= \frac{6}{240}$$

$$= \frac{1}{40}$$

EXERCISES

EXAMPLES 3 and 4 on p. 863 for Exs. 20–21

You randomly choose 2 marbles from the bag described in the example above. Find the probability that both are green if:

20. you replace the first marble.

21. you don't replace the first marble.

13.5 Analyze Surveys and Samples

pp. 871–874

EXAMPLE

You want to determine what type of music is the favorite of students in your grade. You survey every third student from an alphabetical list of students in your grade. You ask each surveyed student, "What is your favorite type of music, classical or country?"

Identify the population and classify the sampling method. Tell whether the question is potentially biased. Explain your answer. If the question is potentially biased, rewrite it so that it is not.

The population is all students in your grade. Because you use the rule "survey every third student," the sample is a systematic sample.

The question is biased, because it does not allow students to choose a type of music other than classical or country. An unbiased question is "What is your favorite type of music?"

EXERCISES

EXAMPLE 1
on p. 871
for Ex. 22

22. **SURVEY** In the example above, suppose you create a questionaire and distribute one to every student in your grade. There is a box in the cafeteria where students can drop off completed questionaires during lunch. Identify the sampling method.

13.6 Use Measures of Central Tendency and Dispersion

pp. 875–878

EXAMPLE

The amounts of snowfall (in inches) in one town for 8 months of the year are listed below. Find the mean, median, and mode(s) of the data. Which measure of central tendency best represents the data?

0.5, 0.5, 1.5, 2.0, 3.5, 4.5, 16.5, 30.5

$$\bar{x} = \frac{0.5 + 0.5 + 1.5 + 2.0 + 3.5 + 4.5 + 16.5 + 30.5}{8} = \frac{59.5}{8} = 7.4375 \text{ inches}$$

The median is the mean of the two middle values, 2.0 and 3.5, or 2.75 inches.

The mode is 0.5 inch.

The median best represents the data. The mean is greater then most of the data values. The mode is less than most of the data values.

EXERCISES

EXAMPLES 1 and 2
on pp. 875–876
for Ex. 23

23. **BASEBALL STATISTICS** The numbers of home runs hit by baseball player Manny Ramirez against several different opposing teams over 3 seasons are 5, 1, 10, 5, 5, 4, 1, 0, 7, 2, 1, 1, 1, 9, 6, 1, 2, 6, 2, 19, 6, and 17.
    a. Find the mean, median, and mode(s) of the data.
    b. Which measure of central tendency best represents the data? *Explain*.

13 CHAPTER REVIEW

13.7 Interpret Stem-and-Leaf Plots and Histograms

pp. 881–885

EXAMPLE

The prices (in dollars) of several books are listed below. Make a stem-and-leaf plot of the prices.

14, 15, 9, 19, 21, 29, 12, 25, 10, 8, 15, 13, 15, 20

STEP 1 **Separate** the data into stems and leaves

Book Prices

Stem	Leaves
0	9 8
1	4 5 9 2 0 5 3 5
2	1 9 0 5

Key: 1 | 4 = $14

STEP 2 **Write** the leaves in increasing order.

Book Prices

Stem	Leaves
0	8 9
1	0 2 3 4 5 5 5 9
2	0 1 5 9

Key: 1 | 4 = $14

EXERCISES

EXAMPLE 1 on p. 881 for Ex. 24

24. EXERCISING The minutes per day that the students in a class spend exercising are listed below. Make a stem-and-leaf plot of the data.

20, 25, 0, 10, 0, 30, 35, 20, 45, 25, 40, 0, 0, 0, 5, 10, 20, 15, 20, 30

13.8 Interpret Box-and-Whisker Plots

pp. 887–892

EXAMPLE

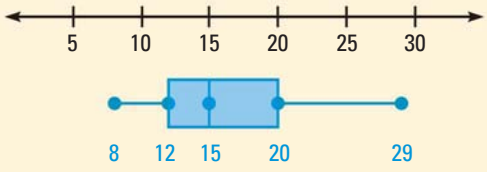
Make a box-and-whisker plot of the book prices in the example above.

Order the data. Then find the median and quartiles.

Upper quartile **Median** = 15 **Lower quartile**

8 9 10 **12** 13 14 15 15 15 19 **20** 21 25 29

Plot the median, the quartiles, the maximum value, and the minimum value below a number line. Draw the box and the whiskers.



EXERCISES

EXAMPLE 1 on p. 887 for Ex. 25

25. EXERCISING Use the data in Exercise 24 to make a box-and-whisker plot of the minutes per day that the students in the class spend exercising.

13 CHAPTER TEST

You roll a number cube. Find (a) the probability that the number rolled is as described and (b) the odds in favor of rolling such a number.

1. a 4

2. an even number

3. a number less than 5

4. a multiple of 3

Evaluate the expression.

5. ${}_7P_2$

6. ${}_8P_3$

7. ${}_6C_3$

8. ${}_{12}C_7$

Tell whether the question can be answered using *combinations* or *permutations*. *Explain* your choice, then answer the question.

9. Eight swimmers participate in a race. In how many ways can the swimmers finish in first, second, and third place?

10. A restaurant offers 7 different side dishes. In how many different ways can you choose 2 side dishes?

In Exercises 11 and 12, refer to a bag containing 12 tiles numbered 1–12.

11. You choose a tile at random. What is the probability that you choose a number less than 10 or an odd number.

12. You choose a tile at random, replace it, and choose a second tile at random. What is the probability that you choose a number greater than 3, then an odd number.

13. **GOVERNMENT PROJECT** City officials want to know whether residents will support construction of a new library. This question appears on the ballot in the citywide election: "Do you support a tax increase to replace the old, deteriorating library with a brand new one?"Is the question potentially biased? *Explain* your answer. If the question is potentially biased, rewrite it so that it is not.

14. **BASKETBALL** The back-to-back stem-and-leaf plot shows the heights (in inches) of the players on a high school's basketball teams.

Basketball Players' Heights

Girls		Boys
9 7 7 6 6 5 3 3	6	9 9 9
3 2 1 1 0	7	0 0 0 2 4 4 6 6 7 7 7 8

Key: 3 | 6 | 9 = 63 in., 69 in.

a. Find the mean, median, and mode(s) of each data set. Which measure of central tendency best represents each data set? *Explain*.

b. Find the range and mean absolute deviation of each data set. Which team's heights are more spread out? *Explain*.

c. Make a box-and-whisker plot of each data set.

d. *Compare* the boys' heights with the girls' heights.

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors, *or*
- solution is without error but incomplete

No Credit
- no solution is given, *or*
- solution makes no sense

SHORT RESPONSE QUESTIONS

PROBLEM

The lengths (in inches) of several goldfish are listed below. Make a box-and-whisker plot of the lengths. Can any of the goldfish lengths be considered outliers? *Explain* why or why not.

8, 5, 4, 5, 4, 5, 4, 3, 4, 8

Below are sample solutions to the problem. Read each solution and the comments in blue to see why the sample represents full credit, partial credit, or no credit.

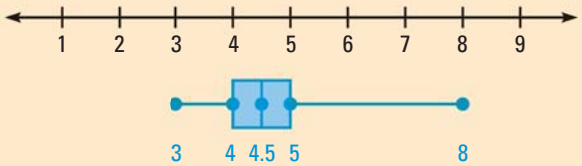
SAMPLE 1: Full credit solution

First, order the lengths from least to greatest.

3, 4, 4, 4, 4, 5, 5, 5, 8, 8

Then, plot the median, the quartiles, the maximum value, and the minimum value below a number line. Draw the box and whiskers.

The box-and-whisker plot is correct, and the student explained how it was drawn.



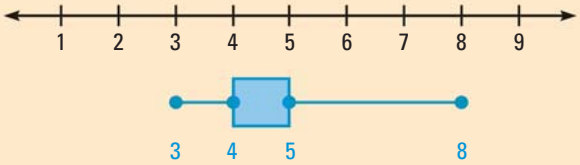
The interquartile range of the goldfish lengths is $5 - 4 = 1$, and 1.5 times the interquartile range is $1.5 \cdot 1 = 1.5$.

The question is answered correctly and includes an explanation.

A length that is less than $4 - 1.5 = 2.5$ would be an outlier. A length that is greater than $5 + 1.5 = 6.5$ would also an outlier. So, the two fish lengths of 8 inches are outliers.

SAMPLE 2: Partial credit solution

The box-and-whisker plot is incorrect. The student has not identified the median.



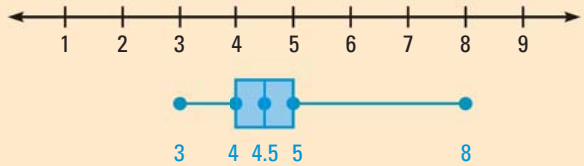
The answer and reasoning are correct.

The interquartile range of the lengths is $5 - 4 = 1$, and $1 \cdot 1.5 = 1.5$.

A length that is less than $4 - 1.5 = 2.5$ is an outlier. A length that is greater than $5 + 1.5 = 6.5$ is an outlier. So, the two fish lengths of 8 inches are outliers.

SAMPLE 3: Partial credit solution

The box-and-whisker plot is correct.



The answer is correct, but the reasoning is incorrect.

The interquartile range of the goldfish lengths is $5 - 4 = 1$.

A length that is less than $4 - 1 = 3$ is an outlier. A length that is greater than $5 + 1 = 6$ is an outlier. So, the two fish lengths of 8 inches are outliers.

SAMPLE 4: No credit solution

There is no box-and-whisker plot. The answer is incorrect.

The value 3 is an outlier because it is a very small goldfish.

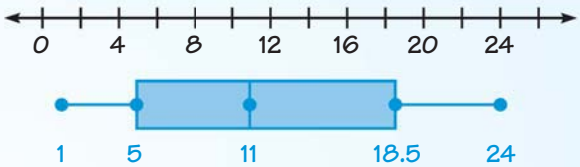
PRACTICE Apply the Scoring Rubric

Score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning.

PROBLEM The number of runs scored by 13 players on a baseball team are listed below. Make a box-and-whisker plot of the data. Can any of the values be considered outliers? *Explain* why or why not.

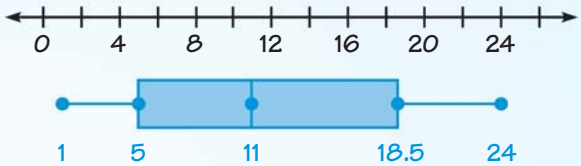
24, 20, 20, 11, 17, 6, 16, 16, 6, 5, 1, 5, 4

1.



There are no outliers in the data set.

2.



The interquartile range is 13.5, and $1.5 \cdot 13.5 = 20.25$. No values are less than $5 - 20.25 = -15.25$ or greater than $18.5 + 20.25 = 38.75$. So, there are no outliers.

13 ★ Standardized TEST PRACTICE

SHORT RESPONSE

1. Your English teacher gives you a list of 5 books that you are required to read over summer vacation. You read the books in a random order.

a. In how many different ways can you read the 5 books?

b. What is the probability that you read the longest book first or second? *Explain* how you found this probability.

2. The median ages (in years) of residents of 13 towns in a county are listed below.

39, 35, 34, 40, 33, 30, 37,
27, 33, 29, 33, 31, 35

a. Make a box-and-whisker plot of the ages.

b. Can any of the ages be considered outliers? *Explain* why or why not.

3. The lengths (in seconds) of songs on one CD are listed below.

136, 249, 434, 136, 299,
227, 270, 270, 46, 254

a. Find the mean, median, and mode(s) of the song lengths.

b. Which measure of central tendency best represents the data? *Explain*.

4. You are ordering a pizza with 3 toppings. There are 8 toppings available.

a. How many possible pizzas with 3 toppings can you order?

b. Did you answer the question in part (a) using combinations or permutations? *Explain* your choice.

5. The prices (in dollars) of several mobile phones sold by one retailer are listed below.

350, 395, 429, 300, 569, 200, 500, 10,
234, 245, 440, 50, 800, 390, 440, 338

a. Make a box-and-whisker plot of the mobile phone prices.

b. Which prices, if any, can be considered outliers? *Explain*.

6. You want to find out what kinds of food items would be most popular to sell to people who attend basketball games at your high school. You decide to conduct a survey.

a. *Describe* how you could choose a representative sample.

b. Write an unbiased question that you could use to collect information on what kinds of food items people would be most likely to purchase during a basketball game. *Explain* why your question is unbiased.

7. The student council has ordered T-shirts for everyone who participated in a recent fundraiser. The table below shows the number of each type of T-shirt ordered. You reach into the box of T-shirts and choose one at random.

	Medium	Large
Long sleeve	9	10
Short sleeve	8	13

a. What is the probability that you choose a medium long-sleeve T-shirt?

b. What is the probability that you choose a medium T-shirt or a long-sleeve T-shirt? *Explain* how this probability is related to the probability you found in part (a).

8. The back-to-back stem-and-leaf plot shows the prices (in dollars) of 15 dinners at two competing restaurants. *Compare* the prices at the two restaurants.

Dinner Prices

Restaurant A		Restaurant B
9 9 9 8	0	
7 7 5 5 2 2 1 0	1	0 2 2 3 5 6 6 8
1 0 0	2	1 2 4 4 5 5 5

Key: 0 | 2 | 1 = \$20, \$21

MULTIPLE CHOICE

9. The odds in favor of an event are 3 : 4. What is the probability of the event?

Ⓐ $\frac{1}{4}$ Ⓑ $\frac{3}{7}$

Ⓒ 75% Ⓓ $\frac{3}{4}$

10. A bag contains 4 red marbles, 3 green marbles, and 5 blue marbles. You randomly choose a marble from the bag. What is the probability that you choose a blue marble?

Ⓐ $\frac{1}{5}$ Ⓑ $\frac{5}{12}$

Ⓒ $\frac{5}{11}$ Ⓓ $\frac{5}{7}$

11. You roll a number cube. What is the probability that you roll a multiple of 2 or a multiple of 3?

Ⓐ $\frac{1}{6}$ Ⓑ $\frac{1}{3}$

Ⓒ $\frac{2}{3}$ Ⓓ $\frac{5}{6}$

GRIDDED ANSWER

12. What is the value of ${}_4P_3$?

13. In how many ways can you arrange the letters in the word BEACH?

14. What is the range of the given data set?

32, 41, 29, 28, 40, 78, 56, 23, 61, 30

15. The stem-and-leaf plot shows the ages (in years) of members of one family. What is the median age?

0	8 9
1	0 4 6 7
2	0
3	9
4	2 3 3 4 5
5	
6	8 9

Key: 0 | 8 = 8 years

EXTENDED RESPONSE

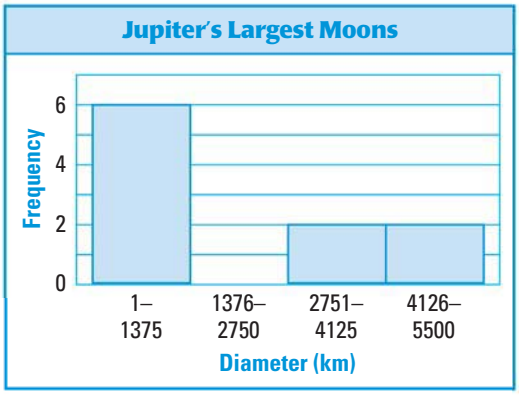
16. A survey asked 500 teenagers where they would like to live. Of those surveyed, 150 teenagers would like to live in a large city. A participant in this survey is chosen at random.

a. What is the probability that the participant would like to live in a large city?

b. What are the odds in favor of the participant's wanting to live in a large city?

c. *Explain* how the probability in part (a) and odds in part (b) are related.

17. The histogram shows the diameters (in kilometers) of Jupiter's ten largest moons.



a. *Describe* the distribution of the data in the histogram. In your description, mention whether the data appear to be spread out or clumped in a certain way.

b. The diameters (in kilometers) of Saturn's ten largest moons are listed below.

97, 209, 256, 536, 560, 764, 2575, 180, 718, 110

Make a histogram of the diameters.

c. *Compare* the distribution of diameters of Jupiter's moons with the distribution of diameters of Saturn's moons.

CUMULATIVE REVIEW Chapters 1–13

Evaluate the expression.

1. $2^4 \cdot 3 - 16 \div 4$ *(p. 8)*

2. $|-125| - 34$ *(p. 80)*

3. $\pm\sqrt{2025}$ *(p. 110)*

Solve the equation.

4. $7 - 2x = 13$ *(p. 141)*

5. $-8x + 15 + 5x = 9$ *(p. 148)*

6. $5(2x + 3) = 4x$ *(p. 154)*

Graph the equation.

7. $x = 7$ *(p. 215)*

8. $y = 2x + 3$ *(p. 244)*

9. $4y - 2x = 1$ *(p. 244)*

Write an equation in slope-intercept form of the line with the given characteristics.

10. passes through $(-2, -8)$ and $(3, -5.5)$ *(p. 292)*

11. slope: -8; passes through $(1, -5)$ *(p. 292)*

Solve the inequality. Graph your solution.

12. $4x - 6 \le 8x - 2$ *(p. 369)*

13. $-2 \le x - 6 < 18$ *(p. 380)*

14. $2x < 6$ *or* $4x \ge 8$ *(p. 380)*

Solve the linear system.

15. $x = 4y + 3$ *(p. 435)*
$2x - 4y = 7$

16. $3x - 7y = 20$ *(p. 451)*
$-11x + 10y = 5$

17. $-9x + 6y = 0$ *(p. 451)*
$-12x + 8y = 5$

Simplify the expression. Write your answer using only positive exponents.

18. $(2x^3)^4 \cdot x^9$ *(p. 489)*

19. $(-9x^3)^2\left(-\frac{1}{4}x^6\right)$ *(p. 489)*

20. $\dfrac{(3x)^{-3}y^3}{x^2y^{-1}}$ *(p. 503)*

Factor the polynomial.

21. $a^2 - 15a - 54$ *(p. 583)*

22. $-3b^2 - 22b - 7$ *(p. 593)*

23. $4f^2 + 4fg + g^2$ *(p. 600)*

24. $p^2(p - 5) + 9(5 - p)$ *(p. 606)*

Solve the equation.

25. $(x + 7)(x - 3) = 0$ *(p. 575)*

26. $9x^2 - 28x + 3 = 0$ *(p. 652)*

27. $8x^2 + 7 = 36x - 9$ *(p. 661)*

28. $\sqrt{x + 8} + 10 = 2$ *(p. 729)*

Find the distance between the two points. *(p. 744)*

29. $(5, 2), (7, 14)$

30. $(-8, 6), (5, 0)$

31. $(2.5, 7), (2.5, -8)$

Find the sum, difference, product, or quotient.

32. $\dfrac{x - 2}{x + 5} \cdot \dfrac{x + 5}{x - 8}$ *(p. 802)*

33. $\dfrac{x^3 - 16x}{x^2 + 3x} \div (x - 4)$ *(p. 802)*

34. $\dfrac{16}{2x^4} \cdot \dfrac{7x^3}{2x}$ *(p. 802)*

35. $\dfrac{2x}{3 - x} + \dfrac{x - 9}{3 - x}$ *(p. 812)*

36. $\dfrac{1}{x + 6} + \dfrac{4x}{x + 6}$ *(p. 812)*

37. $\dfrac{9}{x^2 - 3x} - \dfrac{3}{x - 3}$ *(p. 812)*

Evaluate the expression.

38. ${}_6P_1$ *(p. 851)*

39. ${}_8P_3$ *(p. 851)*

40. ${}_7C_3$ *(p. 856)*

41. ${}_{10}C_6$ *(p. 856)*

42. You roll a number cube. What is the probability that you roll a 5? *(p. 843)*

43. You roll a number cube. What is the probability that you roll a 2 or an even number? *(p. 861)*

44. You choose a number from 1 to 20 at random. What is the probability that you choose a prime number? *(p. 843)*

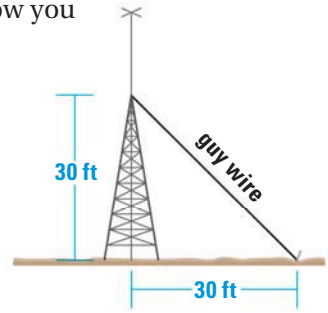
45. You choose a number from 1 to 20 at random. What is the probability that you choose a multiple of 6? *(p. 843)*

46. A bag contains 2 red marbles, 4 green marbles, and 4 blue marbles. You choose one marble at random, put the marble back into the bag, then choose a second marble at random. What is the probability that you choose 2 red marbles? *(p. 861)*

47. **MARATHON** Two runners are training for a marathon. When running a practice distance of 26.2 miles, one runner begins running 6 minutes after the other. The speed of the first runner is 11.4 miles per hour. The speed of the second runner is 12 miles per hour. After how many minutes does the second runner pass the first runner? *(p. 435)*

48. **STONE ARCH** The shape of a stone arch can be modeled by the graph of the equation $y = -0.5x^2 + 4x + 4$ where x is the horizontal distance (in feet) from one end of the arch and y is its height (in feet) above the ground. What is the maximum height of the arch? *Explain* how you found your answer. *(p. 628)*

49. **GUY WIRE** A guy wire supports an antenna tower, as shown at the right. The bottom of the wire is secured in the ground 30 feet from the base of the tower. The top of the wire is secured to the tower at a height of 30 feet above the ground. How long is the wire? Round your answer to the nearest tenth of a foot. *(p. 737)*



50. **HEATING RATES** An electric heater takes 8 minutes to heat an entire apartment to the desired temperature. A wood stove and an electric heater together take 6 minutes to heat the apartment. How many minutes does it take the wood stove alone to heat the apartment to the desired temperature? *(p. 820)*

51. **FLIGHTS** You are traveling from Boston, Massachusetts, to Richmond, Virginia. The prices (in dollars) of airline tickets for different flights between the cities are listed below.

    176, 191, 195, 197, 197, 204, 204, 204, 204,
    204, 206, 206, 206, 206, 206, 217, 217, 221

    a. What is the range of the prices? *(p. 875)*

    b. Make a stem-and-leaf plot of the prices. *(p. 881)*

    c. Make a box-and-whisker plot of the prices. *(p. 887)*

    d. Can any of these prices be considered outliers? *Explain* why or why not. *(p. 887)*